Contents

Preface to the English Edition v

Preface to the Russian Edition vi

From the Editor x

I The Main Boundary Value Problems in the Nonlinear Theory of Shallow Shells 1

1. Results from the Theory of Surfaces 1
2. *S*-Coordinates in Space. Formation of a Shell. Components of Finite Deformation in *S*-Coordinates and Their Simplification . 14
3. The Kirchhoff–Love Hypotheses. Their Mathematical and Mechanical Content. Computation of Deformations of a Shallow Shell Using the Kirchhoff–Love Hypotheses 19
4. Potential Energy of Deformation of a Shallow Shell 26
5. Independent Displacements, Generalized Stresses and the Work of External Forces Under the Kirchhoff–Love Hypotheses 33
6. Boundary Value Problems in Displacements of the Moderate Bending Theory for Shallow Shells 36
7. Boundary Value Problems with Airy Stress Function in the Moderate Bending Theory for Shallow Shells 44
8. Some Remarks on Nonlinear Shallow Shell Theory. A Historical Survey 56

II General Mathematical Questions 60

9. Some General Mathematical Results 60

10. General Mathematical Results (Continued) 73
11. The Function Spaces H_t, $t = 5, 6, 7, 8$. Properties of Their Elements 82
12. The Function Spaces H_κ, $\kappa = 1, 2, 3, 4$. Properties of Their Elements 96

III Topological Methods Applied to Solvability of the Main Boundary Value Problems of the Nonlinear Theory of Shallow Shells in Displacements 107

13. The Generalized Formulation of Boundary Value Problems in Displacements. Reduction to Operator Equations. The Physical Meaning of Generalized Solutions 107
14. Some Properties of the Operators $\mathbf{K}_{t\kappa}$, $\mathbf{G}_{\kappa\kappa}$ 116
15. Computation of the Winding Number of the Vector Field $w - \mathbf{G}_{\kappa\kappa}(w)$ on Spheres of Large Radius in H_κ: Preliminary Lemmas 123
16. Computation of the Winding Number of the Vector Field $\mathbf{w} - \mathbf{G}_{\kappa\kappa}(\mathbf{w})$ on Spheres of Large Radius in H_κ. Solvability of the Main Boundary Value Problems in Displacements 130

IV The Topological Method in the Problem of Solvability of the Main Boundary Value Problems in the Nonlinear Theory of Shallow Shells with an Airy Stress Function 145

17. The Generalized Formulation of the Boundary Value Problems of Shallow Shells with an Airy Stress Function. Reduction to Operator Equations. Physical Interpretation of Generalized Solutions 145
18. Main Properties of the Operators $\mathbf{K}_{9\kappa}(w)$, $\mathbf{G}_\kappa(w)$ 153
19. Computation of the Winding Number of the Vector Field $w - \mathbf{G}_{\kappa(}w)$ on Spheres of Large Radius in H_κ. Solvability of the Main Boundary Value Problems of the Theory of Shallow Shells with an Airy Stress Function 159
20. Differentiability Properties of Generalized Solutions of the Problems $t\kappa$ and 9κ. Conditions for the Existence of Classical Solutions 169

V The Variational Approach to the Problem of Solvability of Boundary Value Problems of Nonlinear Shallow Shell Theory 181

21. The Variational Approach to the Problem of Solvability of Boundary Value Problems of Nonlinear Shallow Shell Theory in Displacements 181
22. The Variational Approach to the Problem of Solvability of Boundary Value Problems of Nonlinear Shallow Shell Theory with an Airy Stress Function 195

VI Numerical-Analytical Methods in the Nonlinear Theory of Shallow Shells **206**

23. Expansion in Powers of a Small-parameter (Nonsingular Solutions) 206
24. Expansion in Powers of a Small-parameter (Singular Solutions). The Liapunov–Schmidt Method 212
25. The Newton–Kantorovich Method 219

VII Direct Methods in the Nonlinear Theory of Shallow Shells **229**

26. Variational Methods for Approximate Solutions of Problems $t\kappa$ ($\kappa = 1, 2, 3, 4$; $t = 5, 6, 7, 8$). The Version of Papkovich . 229
27. The Bubnov–Galerkin–Ritz Method for Approximate Solution of Problems $t\kappa$ ($\kappa = 1, 2, 3, 4$; $t = 5, 6, 7, 8$). The Versions of Mushtari and Vlasov 240
28. Error Estimates for the Bubnov–Galerkin–Ritz (BGR) Method in Some Problems of the Nonlinear Theory of Shallow Shells . 251

VIII Formulation of the Problem of Stability. Global Uniqueness of Solutions. Stiffness of Shells. Well-Posedness Classes **263**

29. Formulation of the Problem of Stability in the Nonlinear Theory of Shallow Shells. Local Uniqueness of Solutions. Conditions for Global Uniqueness 263
30. Physical Stiffness of Shells. Connection with Geometrical Stiffness of the Middle Surface 273
31. Well-Posedness of Problems of the Nonlinear Theory of Shallow Shells: Its Relation to Physical Stability 281

IX Stability in the Large of the Membrane State of a Shallow Shell. Existence of the Lower Critical Value **296**

32. Momentless State of Shells. Passage to the Linearized Problem. Spectral Properties of the Linearized Problem 296
33. Global Stability of Shells in Problems $t\kappa$. Existence of Lower Critical Numbers. Some Estimates for U-Decompositions . . . 306
34. Global Stability of Shells in Problems 9κ. Existence of Lower Critical Values. Some Estimates for U-Decompositions 320
35. Bifurcation of Solutions in a Neighborhood of the Momentless State 325
36. Variational Methods in Global Stability of Shallow Shells . . . 330
37. Some Problems of Global Stability of Plates 336

X A Probabilistic Approach to the Problem of Stability of Shallow Shells **343**

38. A Probabilistic Model of Operations of a Shallow Shell Under Moderate Bending 343

Some Unsolved Problems of the Mathematical Theory of Shells 353

References 355

List of Symbols 375

Index 381