

CONTENTS

<i>Lecturers</i>	ix
<i>Seminar speakers</i>	x
<i>Participants</i>	xi
<i>Préface (French)</i>	xvii
<i>Preface (English)</i>	xxiii
 <i>Course 1. Universalities: from Anderson Localization to Quantum Chaos, by B.L. Altshuler and B.D. Simons</i>	 <i>I</i>
1. Introduction	5
1.1. Impurity states	6
1.2. Impurity bands	7
1.3. Anderson model	9
2. Quantum interference effects in disordered conductors	11
2.1. Waves in random media	11
2.2. Aharonov–Bohm oscillations	13
2.2.1. hc/e	13
2.2.2. $hc/2e$	15
2.3. Variations in random potential	15
2.4. Diffusion	16
2.5. Thouless approach	18
3. Scaling theory of localization	19
4. Perturbation theory of disordered metals	21
4.1. Green functions	21
4.2. Random potential	22
4.3. Diagrammatics	22
4.4. Conductivity	24
4.4.1. Drude conductivity	25
4.4.2. Density correlator and the ladder approximation	27

4.4.3.	Maximally crossed diagrams	28
4.4.4.	Spin scattering	30
4.4.5.	Anomalous magnetoconductance	32
4.4.6.	Inelastic processes	34
4.5.	Quasi-elastic scattering	36
4.6.	Noise and dephasing	38
4.7.	Effects of electron–electron interaction	40
4.8.	Zero-bias anomaly	41
5.	Diffusion modes and fluctuations	42
5.1.	Density of states fluctuations	42
5.2.	Conductance fluctuations	44
5.3.	Parameters of perturbation theory	45
6.	Field theory of localization	45
6.1.	Supersymmetry method	46
6.1.1.	Functional integral	46
6.1.2.	Ensemble averaging	48
6.1.3.	Hubbard–Stratonovitch transformation	49
6.1.4.	Saddle-point equation	50
6.1.5.	Non-linear σ -model	51
6.1.6.	Magnetic fields	53
6.1.7.	Spin-orbit scattering	54
6.1.8.	Universality classes	54
6.2.	Renormalization group and the $2 + \epsilon$ expansion	55
7.	Zero-dimensional σ -model	57
7.1.	Aharonov–Bohm flux	59
7.2.	Random matrix theory	65
8.	Ergodicity, universality, and quantum chaos	69
8.1.	Quantum billiards	70
8.2.	Numerics	70
8.3.	Hard and soft chaos	72
8.4.	Periodic orbit theory	73
9.	T-Violation and the crossover between ensembles	74
10.	Beyond level correlations	76
10.1.	Continuity relation	76
10.2.	Applications	80
10.2.1.	Dynamical echo	80
10.2.2.	Oscillator strengths	81
10.2.3.	Magnetoconductivity	83
10.2.4.	Dielectric response of a periodic irregular structures	84
10.3.	Wavefunction correlations	85
11.	Level dynamics and the matrix field theory	86
11.1.	Supersymmetry approach to the matrix field theory	89
12.	Discussion	93
	References	94

<i>Course 2. Experimental Signatures of Phase Coherent Transport, by H. Bouchiat</i>	99
1. Introduction	103
1.1. The phase coherence length	103
1.2. Outline of the course	105
2. Manifestation of phase coherence in macroscopic samples: from weak to strong localization	105
2.1. Weak localization	105
2.1.1. Coherent backscattering of electrons	105
2.1.2. Magnetoconductance: signature of time reversal symmetry breaking	107
2.1.3. Temperature dependence of the phase coherence time	109
2.1.4. Weak localization as an experimental tool for measuring microscopic parameters in solid state physics	112
2.2. Metal–insulator transition	113
3. Mesoscopic regime: transport at the scale of the phase coherence length	116
3.1. Definitions of the conductance of a phase coherent metallic sample	116
3.1.1. Conductance and absorption	117
3.1.2. Conductance and transmission	117
3.1.3. Conductance and sensitivity to boundary conditions	118
3.2. Universal fluctuations and quantum oscillations of the conductance of connected samples	119
3.2.1. Physical origin	119
3.2.2. Magneto-fingerprint of a mesoscopic sample and Aharonov–Bohm oscillations	120
3.2.3. Fluctuations and quantum oscillations in Mosfets and doped semiconductors	122
3.2.4. Measurements at length scale below L_ϕ : non-local effects	125
3.2.5. Fluctuations induced by the motion of impurities: $1/f$ and telegraphic noises	126
3.3. Energy and temperature averaging of conductance fluctuations and h/e oscillations	130
3.3.1. Relevance of the Thouless energy scale $E_c = hD/L^2$	130
3.3.2. Effect of the coherence length L_ϕ	132
3.3.3. Application to the study of magnetic impurities	133
4. Properties of isolated or quasi-isolated mesoscopic systems	135
4.1. Persistent currents and orbital magnetism	135
4.1.1. Definition, orders of magnitude	135
4.1.2. Experiments	137
4.2. AC conductance of Aharonov–Bohm rings	140
4.2.1. Principle and motivation of the experiments.	140
4.2.2. Experimental results	141

5. Conclusion	143
References	143

Course 3. Recent Experiments on Multiple Scattering and Localization of Light, by G. Maret 147

1. Introduction	151
1.1. Single scattering	151
1.1.1. Rayleigh scattering, ($R \ll \lambda$):	151
1.1.2. Rayleigh–Debye–Gans scattering ($kR m - 1 \ll 1$):	152
1.1.3. Mie scattering (any kR , any m):	153
1.2. Random walks and classical diffusion	154
1.3. Classical wave interferences, speckles	155
2. Coherent backscattering and localization	156
2.1. Weak localization and enhanced backscattering	156
2.1.1. Angular dependence	158
2.1.2. Absorption and finite sample size	160
2.1.3. Internal reflections	161
2.1.4. Enhancement factor	161
2.2. Weak positional correlations of scatterers	162
2.3. Polarization	163
2.4. Magneto-optical Faraday effect	164
2.5. Long range speckle correlations	165
2.6. Nonlinear media	167
2.7. Attempts and difficulties to observe 3D localization of light	168
3. Applications	169
3.1. Dynamic correlation function of the multiple scattering intensity	169
3.2. Brownian motion of colloidal scatterers	170
3.2.1. Particle sizing	171
3.2.2. Interparticle correlations	172
3.2.3. Short time dynamics	173
3.3. Convective motion	173
3.4. Recent applications of “diffusing wave spectroscopy”	174
3.5. Imaging through turbid media	174
3.6. Medical applications	175
4. Conclusion: light versus electrons	175
References	176

Course 4. Anderson Insulators, by Y. Imry 181

1. Introduction, basic concepts	185
2. The Thouless picture and the scaling theory of localization	190

3. Transport in the localized phase	199
4. Magnetoconductance in the hopping regime, the effect of the magnetic field on localization	202
5. Frequency-dependent Hall effect at zero temperature in the Anderson insulator	210
6. Orbital magnetization in the hopping regime	215
7. The magnetic moment and the Hall effect in the presence of the electron–phonon interaction	218
Acknowledgements	224
References	225

Seminar 1. Localization in Disordered Systems, by A. Shalgi and Y. Imry 229

1. Anderson’s theory of localization	231
1.1. Introduction	231
1.2. Anderson criterion for localization	232
1.3. The Locator expansion	235
1.4. The role of the effective potential	237
1.5. The analytical properties of the “self-energy”	239
1.6. Convergence of Δ_0 in a probability sense	242
2. Anderson localization in the presence of a magnetic field	245
2.1. Magnetic fields — general discussion	245
2.2. The self-energy in the presence of a magnetic field	246
2.3. Convergence of Δ_0 in the case of a magnetic field	247
3. The localization length	248
3.1. General discussion	248
3.2. Derivation of the localization length from the “self-energy”	249
3.3. Divergence of the localization length near the Anderson transition	253
3.4. The localization length in a magnetic field	254
4. Summary	255
Acknowledgements	255
Appendix A. Probability distribution of a sum and of a single largest term	255
References	257

Seminar 2. Interferences in Disordered Mesoscopic Insulators, by F. Ladieu and M. Sanquer 259

1. Introduction	260
2. Interference effects in Anderson Insulators at $T = 0$	261
3. Experiment	262
4. Variable Range Hopping regime and incoherent mesoscopics	262
5. Magnetic field effects	265

6. Conclusion	266
References	267

Seminar 3. Quasicrystals: Metal–Insulator Transition in Highly Ordered Metallic Phase, by C. Berger 269

1. Introduction	270
2. High structural quality quasicrystals, quasiperiodicity and approximants	270
3. Low conductivity values	271
4. Proximity of a metal–insulator transition, pseudo-gap	273
5. Hume–Rothery picture	275
6. Beyond a classical picture	275
7. Conclusion	277
References	277

Course 5. Quantum Transport in Semiconductor–Superconductor Microjunctions, by C.W.J. Beenakker 279

1. Introduction	283
2. Scattering theory	286
3. Three simple applications	292
3.1. Quantum point contact	292
3.2. Quantum dot	293
3.3. Disordered junction	295
4. Weak localization	297
5. Reflectionless tunneling	300
5.1. Numerical simulations	300
5.2. Scaling theory	303
5.3. Double-barrier junction	308
5.4. Circuit theory	311
6. Universal conductance fluctuations	316
7. Shot noise	318
8. Conclusion	320
References	322

Course 6. Transport Theory of Mesoscopic Systems: Application to Ballistic Transport, by A.D. Stone 325

1. Introduction	329
-----------------	-----

1.1.	Background	329
1.2.	Novel properties of mesoscopic conductors	329
2.	Sample-specific theory	334
2.1.	Conductance coefficients	334
2.2.	The scattering approach	334
2.3.	Time-reversal symmetry constraints on g	336
2.4.	Resistance quantization and negative resistance	337
3.	Sample-specific linear response theory	339
3.1.	Model hamiltonian for mesoscopic conductors	339
3.2.	Kubo non-local conductivity tensor	341
3.3.	Conductance coefficients in linear response	343
4.	Ballistic transport phenomena	347
4.1.	Novel phenomena in high-mobility microstructures	347
4.2.	Semiclassical formalism	349
4.3.	Classical two-probe conductance	352
4.4.	Classical effects in ballistic junctions	353
4.5.	Quantum ballistic effects	356
4.6.	Ballistic weak localization: coherent backscattering	358
4.7.	Ballistic weak localization: field dependence	360
4.8.	Problems with unitarity	362
4.9.	Ballistic conductance fluctuations	364
4.10.	Distinguishing regular from chaotic scattering	365
4.11.	Other aspects of chaotic ballistic transport	367
4.12.	Summary	368
	Acknowledgements	368
	References	369

<i>Course 7.</i>	<i>Semiclassical Quantization of Chaotic Billiards – a Scattering Approach, by U. Smilansky</i>	<i>373</i>
------------------	---	------------

1.	Introduction	377
2.	Classical billiards	379
2.1.	Billiards and maps	379
2.2.	Area preserving maps	382
2.3.	Periodic orbits in billiards	387
3.	Exact quantization – a scattering approach	390
3.1.	Scattering into waveguides	391
3.2.	Scattering in the plane	394
4.	The semiquantal approximation	399
4.1.	The semiquantal restriction	399
4.2.	The semiquantal secular function	402
5.	The semiclassical approximation	405

5.1. The mean spectral density	405
5.2. The semiclassical S matrix	406
5.3. Semiclassical quantization of maps	409
5.4. Computation of $[\text{Tr}(U^n)]_{\text{scl}}$	415
6. The semiclassical spectral functions	419
6.1. The semiclassical spectral density	419
6.2. A numerical check of the semiclassical spectral density	420
6.3. Bouncing ball contributions to the spectral density	422
6.4. Quantization of an integrable billiard – the circle	423
6.5. The semiclassical spectral ζ function	425
7. Summary and conclusions	430
8. Acknowledgements	431
References	431

Course 8. Theory of Random Matrices: Spectral Statistics and Scattering Problems, by P.A. Mello 435

1. Introduction	439
2. Random matrices and spectral statistics	441
2.1. The concepts of information and entropy of a statistical distribution	444
2.2. An ensemble of real symmetric matrices	447
2.3. Universality classes	450
2.4. The Gaussian Ensembles (GE)	450
2.5. Matrix ensembles with a specified density	456
3. Random matrices in scattering problems	457
3.1. Scattering described by an equilibrated component. The circular ensembles	457
3.2. Scattering described by a prompt and an equilibrated component	463
3.2.1. The one-lead, one-channel case	463
3.2.2. The multichannel case	467
3.3. Scattering in disordered media. Diffusion processes and localization	469
3.3.1. The isotropic model: a global approach	470
3.3.2. The evolution of the statistical distribution: a local approach	472
Appendix A. Scattering theory	477
A.1. A survey of quantum-mechanical scattering theories	477
A.1.1. Scattering formalisms	477
A.1.2. Statistical scattering	479
A.2. Simple examples of quantum-mechanical scattering problems	480
A.2.1. A 1×1 scattering matrix: illustration of the concepts of unitarity and analytic structure in the complex energy plane	480
A.2.2. A 2×2 scattering matrix and the corresponding transfer matrix	483
Appendix B. Verification of the central-limit theorem for the 1×1 K matrix	485

Appendix C. The Fokker–Planck equation for disordered systems	487
References	489
 <i>Course 9. Basic Features of Efetov’s Supersymmetry Approach, by Y.V. Fyodorov</i>	 493
1. Introduction	497
1.1. Mathematical prerequisites	498
2. Warming up: Wigner semicircle by supersymmetry approach	502
3. DOS–DOS correlation for the Gaussian Unitary Ensemble	507
3.1. Guiding example: a toy model	511
3.2. Full problem revisited	515
4. Beyond the Random Matrix theory: an overview of the nonlinear σ -model	519
5. Acknowledgements	528
Appendix A. The parametrization of the matrices $\hat{T}$	528
References	531
 <i>Course 10. Fermi Liquids and Non-Fermi Liquids, by H.J. Schulz</i>	 533
1. Introduction	537
2. Fermi liquids	538
2.1. The Fermi gas	538
2.2. Landau’s theory: phenomenological	539
2.2.1. Basic hypothesis	539
2.2.2. Equilibrium properties	541
2.3. Conclusion	545
3. Renormalization group for interacting fermions	546
3.1. One dimension	547
3.2. Two and three dimensions	552
4. Luttinger liquids	556
4.1. Bosonization for spinless electrons	557
4.2. Spin-1/2 fermions	563
4.2.1. Spin–charge separation	565
4.2.2. Physical properties	566
4.2.3. Long-range interactions	569
4.2.4. Persistent current	570
5. Transport in a Luttinger liquid	571
5.1. Conductance and conductivity	572
5.2. Edge states in the quantum Hall effect	573
5.3. Single barrier	575
5.4. Random potentials and localization	578

6. The Bethe ansatz: a pedestrian introduction	582
6.1. The Heisenberg spin chain	583
6.1.1. Bethe's solution	583
6.1.2. The Heisenberg antiferromagnet	586
6.1.3. The Jordan–Wigner transformation and spinless fermions	588
6.2. The Hubbard model	589
6.2.1. The exact solution	590
6.2.2. Luttinger liquid parameters	592
6.2.3. Transport properties and the metal–insulator transition	595
6.2.4. Spin–charge separation	597
7. Conclusion and outlook	598
Acknowledgments	599
References	599

Course 11. Single Electron Phenomena in Metallic Nanostructures, by M. Devoret, D. Esteve and C. Urbina 605

1. Foreword	609
2. General overview of single charge phenomena	609
2.1. Introduction	609
2.2. Basic principles of single electron transfer	610
2.3. Single electron effects: a brief history	614
2.4. The single electron transistor	615
2.5. Controlled transfer of charge flowing in an external circuit	619
2.6. Metrological applications	624
2.7. Single Cooper pair transfer	625
2.8. Future prospects	629
3. Quantum dynamics of tunnel junction circuits	629
3.1. Introduction	629
3.2. Conduction in tunnel junction circuits	630
3.3. Description of the electromagnetic environment	632
3.4. The total electrostatic energy E	634
3.5. Quantum mechanical treatment of the junction coupled to its electromagnetic environment	635
3.6. Calculation of tunnel rates	640
3.7. Properties of $P(E)$	643
3.8. Application of the theory to particular cases of junction environments	644
3.8.1. Ohmic case	644
3.8.2. Case of an electron box with a non ideal voltage bias	644
3.8.3. Single mode case	645
4. Single Cooper pairs	646
4.1. Basic elements of the BCS theory	647

4.2. Extension of the BCS theory to an isolated superconductor	651
4.3. Partition function of the superconducting electron box	652
Acknowledgements	656
References	656

Course 12. Introduction to the Physics of the Quantum Hall Regime, by A.H. MacDonald 659

1. What is the quantum Hall effect?	663
2. Drude theory of magnetotransport	665
3. Free 2D electron in a magnetic field	667
3.1. Classical solution	667
3.2. Quantum solution	668
3.3. Useful identities for symmetric-gauge free-particle eigenfunctions	671
4. Incompressibility	674
4.1. What is incompressibility?	674
4.2. Incompressibility implies quantization	675
5. Integer quantum Hall regime	679
6. Incompressibility at fractional filling factors	681
6.1. Two body problem: Haldane pseudopotentials	682
6.2. Some properties of Haldane pseudopotentials	684
6.3. Incompressibility in the hard-core model and Laughlin wavefunctions	685
7. Laughlin state properties	687
7.1. Fractionally charged quasiparticles	687
7.2. Plasma analogy	690
8. Chern–Simons–Landau–Ginzburg theory	691
8.1. Anyons	691
8.2. Statistical transmutation	692
8.3. Quasiparticle statistics	693
8.4. Digression: lowest Landau level density matrices	694
8.5. Boson off-diagonal long-range order	695
9. Collective modes of incompressible states	697
9.1. Collective modes at zero magnetic field	697
9.2. Correlation function moments in the fractional Hall regime	699
9.3. Projected static structure factor	701
9.4. Magnetorotons	702
10. Hierarchy states	705
10.1. Bosonization	706
10.2. Particle–hole symmetry	709
10.3. Correlation factors	711
10.4. Classical hierarchies	712
10.5. Neoclassical hierarchies	713
10.6. Overview on hierarchy states	715

11. What's not here	716
References	717

Seminar 4. Quantum Transport in the Integer and Fractional Quantum Hall Effect Regime, by D.C. Glattli and C. Pasquier 721

1. Experimental system	722
2. The non-interacting picture of IQHE edge states	723
3. An interacting picture of edge states for IQHE-FQHE	726
References	730

Seminar 5. Ideal Anyons, by S. Ouvry 731

1. Introduction: the Braid group	732
2. Quantum Mechanics: the Anyon model	733
3. Chern–Simons formulation	734
4. Exact results	734
5. Mean field approximation	736
6. Perturbative approach	736
7. Strong magnetic field	738
References	739

Course 13. Adiabatic Quantum Transport, by J.E. Avron 741

Overview	745
1. Manifolds and their calculus	746
1.1. Manifolds	746
1.2. Forms	748
1.3. The exterior derivative	749
1.4. Riemannian metric	749
1.5. Scalar products	749
1.6. Hodge $*$	750
1.7. Cohomology	750
1.8. Harmonic forms	750
1.9. Integration	751
1.10. Homology	751
1.11. Periods	752
2. Aharonov–Bohm fluxes	752
2.1. Summary	752
2.2. Fluxes	752

2.3.	Gauge fixing	754
2.4.	Harmonic gauge	754
2.5.	Singular gauge	755
2.6.	Pure gauge fields	756
2.7.	The flux torus	756
2.8.	A Riemann metric	756
3.	Magnetic fields	757
3.1.	Dirac quantization	757
4.	Schrödinger operators on graphs	758
4.1.	Summary	758
4.2.	The kinetic energy	758
4.3.	Spectral analysis	759
5.	Schrödinger operators on manifolds	761
5.1.	Summary	761
5.2.	Landau Hamiltonians	761
5.3.	Periodic boundary conditions	762
5.4.	Magnetic translation	763
5.5.	Automorphic factors	763
6.	Spectral properties of Landau Hamiltonians	764
6.1.	Summary	764
6.2.	The flat torus	764
6.3.	Riemann surfaces	764
6.4.	Index and heat kernel	765
7.	Currents	766
7.1.	Summary	766
7.2.	Classical	766
7.3.	Loop currents	767
7.4.	The current operator	767
8.	Adiabatic theorems	768
8.1.	Summary	768
8.2.	Hilbert space projections	768
8.3.	The adiabatic setting	769
8.4.	Adiabatic evolution	769
8.5.	Comparison of dynamics	770
9.	The adiabatic curvature	770
9.1.	Parallel transport and Riemannian geometry	770
9.2.	The canonical connection	771
9.3.	Curvature	772
9.4.	Projections of arbitrary rank	773
9.5.	Miscellaneous formulas for the curvature	773
9.6.	General properties of the curvature	774
10.	Constant curvature	775
10.1.	Summary	775
10.2.	Curvature and quantum numbers	775

Contents

10.3. Phase space translations	775
10.4. Landau Hamiltonians on tori	776
10.5. Curvature and gauge transformations	777
10.6. Area of triangles	777
10.7. Curvature for Landau levels	777
11. Conductances	778
11.1. Summary	778
11.2. Setting	778
11.3. Charge transport and adiabatic curvature	779
11.4. Covariance	781
12. Chern numbers	782
12.1. Summary	782
12.2. Chern number of a complex line bundle on S^2	782
12.3. Chern numbers for the flux torus	783
12.4. Diophantine equations	783
13. Counting electrons	784
13.1. Summary	784
13.2. Landau levels	784
13.3. Comparing dimensions	785
13.4. Compact operators	786
13.5. Index	786
13.6. Projection related by unitaries	787
13.7. Homogeneous systems	788
Acknowledgements	789
References	790

