



# Contents

<b>Preface</b>	vii
<b>Conventions and Symbols</b>	xvii
<b>1 Unconstrained Optimization</b>	1
<b>1.1 Optimality Conditions</b>	1
1.1.1 First- and Second-Order Necessary Conditions	3
1.1.2 Sufficient Conditions	11
1.1.3 The Convex Case	13
<b>1.2 Algorithm Models and Convergence Conditions I</b>	15
1.2.1 Geometry of Descent Methods	16
1.2.2 Basic Algorithm Models	18
1.2.3 The Wolfe and Polak-Sargent-Sebastian Theorems	28
1.2.4 A Trust Region Model	35
1.2.5 Algorithm Implementation Theory	40
1.2.6 Rate of Convergence of Sequences	47
1.2.7 Algorithm Efficiency	53
1.2.8 Notes	54
<b>1.3 Gradient Methods</b>	56
1.3.1 Method of Steepest Descent	56
1.3.2 Armijo Gradient Method	58
1.3.3 Projected Gradient Method	66
1.3.4 Notes	70
<b>1.4 Newton's Method</b>	70
1.4.1 The Local Newton Method	70
1.4.2 Global Newton Method for Convex Functions	76
1.4.3 Discrete Newton Method	79
1.4.4 Global Newton Method for General Functions	82
1.4.5 The Iterated Newton Method	83
1.4.6 Notes	87

<b>1.5</b>	<b>Methods of Conjugate Directions</b>	<b>87</b>
1.5.1	Decomposition of Quadratic Functions	88
1.5.2	Methods of Conjugate Gradients	91
1.5.3	Formal Extension to General Functions	94
1.5.4	The Polak-Ribière Conjugate Gradient Algorithm	95
1.5.5	The Fletcher-Reeves Conjugate Gradient Method	99
1.5.6	Partial Conjugate Gradient Methods	102
1.5.7	Notes	103
<b>1.6</b>	<b>Quasi-Newton Methods</b>	<b>104</b>
1.6.1	The Variable Metric Concept	105
1.6.2	Secant Methods	107
1.6.3	Symmetric Rank-One Updates	111
1.6.4	Symmetric Rank-Two Updates	115
1.6.5	Finite Convergence on Quadratic Functions	117
1.6.6	Global Convergence on Convex Functions	124
1.6.7	Notes	137
<b>1.7</b>	<b>One-Dimensional Optimization</b>	<b>138</b>
1.7.1	Secant Method Based on Cubic Interpolation	139
1.7.2	The Golden Section Search	146
1.7.3	Method of Sequential Quadratic Interpolations	149
1.7.4	Notes	157
<b>1.8</b>	<b>Newton's Method for Equations and Inequalities</b>	<b>157</b>
1.8.1	Mangasarian - Fromowitz Constraint Qualification	158
1.8.2	The Local Newton Algorithm	161
1.8.3	Global Newton Method	166
1.8.4	Notes	166
<b>2</b>	<b><i>Finite Min-Max and Constrained Optimization</i></b>	<b>167</b>
<b>2.1</b>	<b>Optimality Conditions for Min-Max</b>	<b>168</b>
2.1.1	First-Order Conditions	169
2.1.2	Optimality Functions	172
2.1.3	Second-Order Conditions	178
2.1.4	Notes	185
<b>2.2</b>	<b>Optimality Conditions for Constrained Optimization</b>	<b>185</b>
2.2.1	First-Order Optimality Conditions for <b>ICP</b>	185
2.2.2	An Optimality Function for <b>ICP</b>	190
2.2.3	Second-Order Conditions for <b>ICP</b>	193
2.2.4	First-Order Optimality Conditions for <b>IECP</b>	197
2.2.5	Second-Order Optimality Conditions for <b>IECP</b>	204
2.2.6	Notes	214
<b>2.3</b>	<b>Algorithm Models and Convergence Conditions II</b>	<b>215</b>

2.3.1	Algorithm Models for <b>ICP</b>	215
2.3.2	Algorithm Models for <b>IECP</b>	219
2.3.3	Notes	222
<b>2.4</b>	<b>First-Order Min-Max Algorithms</b>	<b>222</b>
2.4.1	The PPP Min-Max Algorithm	222
2.4.2	Rate of Convergence of the PPP Algorithm	224
2.4.3	Algorithms for Search Direction Computation	227
2.4.4	Quadratic Convergence to a Haar Point	237
2.4.5	Box-Constrained Min-Max Algorithm	242
2.4.6	A Barrier Function Method	244
2.4.7	Notes	248
<b>2.5</b>	<b>Newton's Method for Min-Max Problems</b>	<b>250</b>
2.5.1	The Local Newton Method	251
2.5.2	The Global Newton Method	255
2.5.3	Notes	258
<b>2.6</b>	<b>Phase I - Phase II Methods of Centers</b>	<b>259</b>
2.6.1	Min-Max-Type Phase I - Phase II Methods	260
2.6.2	Rate of Convergence	264
2.6.3	A Barrier Function Method	274
2.6.4	Notes	279
<b>2.7</b>	<b>Penalty Function Algorithms</b>	<b>280</b>
2.7.1	Basic Theory of Penalty Functions	281
2.7.2	Exact Penalty Functions	291
2.7.3	Exact Penalty Function Algorithms	303
2.7.4	Notes	311
<b>2.8</b>	<b>Augmented Lagrangian Methods</b>	<b>315</b>
2.8.1	Problems with Equality Constraints	315
2.8.2	Problems with Mixed Constraints	324
2.8.3	Notes	333
<b>2.9</b>	<b>Sequential Quadratic Programming</b>	<b>333</b>
2.9.1	Wilson's Method	334
2.9.2	Pang's Method	339
2.9.3	The Local Maratos-Mayne-Polak Method for (2)	344
2.9.4	Global MMP Algorithm for (2)	354
2.9.5	The Maratos-Mayne-Polak-Pang Method for (1)	359
2.9.6	Notes	366
<b>3</b>	<b>Semi-Infinite Optimization</b>	<b>368</b>
<b>3.1</b>	<b>Optimality Conditions for Semi-Infinite Min-Max</b>	<b>369</b>
3.1.1	First-Order Optimality Conditions for <b>SMMP</b>	369
3.1.2	An Optimality Function for <b>SMMP</b>	372

3.1.3	Second-Order Conditions for <b>SMMP</b>	374
3.1.4	Notes	378
3.2	<b>Optimality Conditions for Constrained Semi-Infinite Optimization</b>	378
3.2.1	First-Order Optimality Conditions for <b>SICP</b>	379
3.2.2	An Optimality Function for <b>SICP</b>	381
3.2.3	Second-Order Conditions for <b>SICP</b>	382
3.2.4	First-Order Optimality Conditions for <b>SIECP</b>	385
3.2.5	Second-Order Conditions for <b>SIECP</b>	387
3.2.6	Notes	389
3.3	<b>Theory of Consistent Approximations</b>	389
3.3.1	Epi-convergence and Optimality Functions	390
3.3.2	Penalty Functions	400
3.3.3	Master Algorithm Models	401
3.3.4	Notes	418
3.4	<b>Semi-Infinite Min-Max Algorithms</b>	418
3.4.1	Consistent Approximations	419
3.4.2	Algorithms Based on Algorithm Models 3.3.12 and 3.3.17	423
3.4.3	PPP Rate-Preserving Min-Max Algorithm	426
3.4.4	Newton Rate-Preserving Min-Max Algorithm	431
3.4.5	Method of Outer Approximations	436
3.4.6	Notes	444
3.5	<b>Algorithms for Inequality-Constrained Semi-Infinite Optimization</b>	445
3.5.1	Consistent Approximations	446
3.5.2	Algorithms Based on Algorithm Models 3.3.14 and 3.3.20	449
3.5.3	Method of Outer Approximations	460
3.5.4	Notes	465
3.6	<b>Algorithms for Semi-Infinite Optimization with Mixed Constraints</b>	466
3.6.1	Consistent Approximations	467
3.6.2	Method of Outer Approximations	469
3.6.3	An Exact Penalty Function Algorithm	471
3.6.4	Notes	481
<b>4</b>	<b><i>Optimal Control</i></b>	<b>482</b>
4.1	<b>Canonical Forms of Optimal Control Problems</b>	482
4.1.1	Properties of Defining Functions	486
4.1.2	Transcription into Canonical Form	493
4.1.3	Numerical Integration	494

<b>4.2</b>	<b>Optimality Conditions for Optimal Control</b>	495
4.2.1	Unconstrained Optimal Control	497
4.2.2	Min-Max Optimal Control	502
4.2.3	Optimal Control with Inequality Constraints	511
4.2.4	Optimal Control with Equality Constraints	515
4.2.5	Optimal Control with Equality and Inequality Constraints	529
4.2.6	Notes	532
<b>4.3</b>	<b>Algorithms for Unconstrained Optimal Control</b>	534
4.3.1	Consistent Approximations	535
4.3.2	Problem Reformulation on $\mathbb{R}^n \times \mathbb{R}^{mN}$	541
4.3.3	Algorithms Based on Master Algorithm Model 3.3.12	544
4.3.4	Algorithms Based on Master Algorithm Model 3.3.17	546
4.3.5	Algorithms Based on Master Algorithm Model 3.3.20	548
4.3.6	Implementation of Newton's Method	556
4.3.7	Notes	560
<b>4.4</b>	<b>Min-Max Algorithms for Optimal Control</b>	562
4.4.1	Consistent Approximations	563
4.4.2	Problem Reformulation on $\mathbb{R}^n \times \mathbb{R}^{mN}$	573
4.4.3	Algorithms Based on Master Algorithm Model 3.3.12	575
4.4.4	Algorithms Based on Master Algorithm Model 3.3.17	579
4.4.5	Algorithms Based on Master Algorithm Model 3.3.20	583
4.4.6	Method of Outer Approximations	587
4.4.7	Notes	589
<b>4.5</b>	<b>Algorithms for Problems with State Constraints I: Inequality Constraints</b>	589
4.5.1	Consistent Approximations	590
4.5.2	Problem Reformulation on $\mathbb{R}^n \times \mathbb{R}^m$	594
4.5.3	Algorithms Based on Master Algorithm Model 3.3.14	596
4.5.4	Algorithms Based on Master Algorithm Model 3.3.27	602
4.5.5	Method of Outer Approximations	606
4.5.6	Notes	608
<b>4.6</b>	<b>Algorithms for Problems with State Constraints II: Equality Constraints</b>	609
4.6.1	Consistent Approximations	610
4.6.2	An Exact Penalty Function Algorithm	621
4.6.3	Notes	630
<b>4.7</b>	<b>Algorithms for Problems with State Constraints III: Equality and Inequality Constraints</b>	630
4.7.1	Consistent Approximations	631
4.7.2	An Exact Penalty Function Algorithm	637
4.7.3	Notes	643

<b>5</b>	<b><i>Mathematical Background</i></b>	646
5.1	<b>Results from Functional Analysis</b>	646
5.1.1	Real Normed Spaces	646
5.1.2	Properties of Continuous Functions	651
5.1.3	Derivatives and Expansion Formulas	655
5.1.4	Directional Derivatives and Subgradients	660
5.1.5	The Implicit Function Theorem	664
5.1.6	Notes	665
5.2	<b>Convex Sets and Convex Functions</b>	665
5.2.1	Convex Sets	666
5.2.2	Convex Functions	668
5.3	<b>Properties of Set-Valued Functions</b>	676
5.3.1	Outer and Inner Semicontinuity	676
5.3.2	Notes	681
5.4	<b>Properties of Max Functions</b>	682
5.4.1	Maximum Theorems	682
5.4.2	Directional Derivatives and Subgradients	685
5.4.3	A Mean-Value Theorem	694
5.5	<b>Minimax Theorems</b>	696
5.5.1	Duality and Discrete Minimax Theorems	696
5.5.2	The von Neumann Theorem	703
5.5.3	Notes	709
5.6	<b>Differential Equations</b>	709
5.6.1	Existence, Uniqueness, and Boundedness of Solutions	711
5.6.2	Lipschitz Continuity and Differentiability of Solutions	714
5.6.3	Discrete-Time Approximations	721
5.6.4	Bounds on Approximation Errors	736
5.6.5	Notes	742
	<b><i>Bibliography</i></b>	743
	<b><i>Index</i></b>	773

