

## CONTENTS

<i>Contributors</i>	ii
<i>Participants</i>	ix
<i>Preface (in French and English)</i>	xiii
<i>Course 1. Dislocations and walls in crystals, by Jacques Friedel</i>	1
0. Introduction	5
1. The concept of order in condensed matter	8
1.1. Atomic order	8
1.1.1. Perfect crystals	8
1.1.2. Weak disorder in crystal structures	11
1.1.3. Classical liquids	14
1.1.4. Amorphous states	15
1.1.5. Intermediary cases	18
1.2. Electronic order	22
1.2.1. Mott localisation	22
1.2.2. Magnetism	25
1.2.3. Electron–phonon couplings	31
1.3. Strong perturbations of the order	32
1.3.1. A special case: critical fluctuations near a phase change	32
1.3.2. General case of first order transition. Hysteresis	33
1.3.3. Defects created by other efforts	40
2. Walls in crystals	41
2.1. Geometry	41
2.1.1. Macroscopic parameters	41
2.1.2. Microscopic parameters	42
2.1.3. Small angle boundaries	42
2.1.4. Common large angle boundaries	45
2.1.5. Special low energy boundaries. Coherent twins	45

2.1.6. Approximate special low angle boundaries. Approximately coherent twins	46
2.1.7. Stacking faults	46
2.1.8. Epitaxy	47
2.1.9. Reshuffling	49
2.2. Interfacial tension	50
2.2.1. Measurements	50
2.2.2. Estimates	51
2.2.3. Computations. General remarks	51
2.2.4. sp metals	52
2.2.5. Transitional (d) metals	61
2.2.6. sp covalents	68
2.2.7. Ionic solids	72
2.2.8. Entropy of vibration	73
2.3. Mobility of walls	76
2.3.1. Slip of a simple magnetic wall of classical spins	76
2.3.2. Motion of walls in crystals	83
3. Dislocations	87
3.1. Infinitesimal dislocations in continuous homogeneous and isotropic macroscopic solids	87
3.2. Dislocations in continuous homogeneous, but anisotropic solids	96
3.2.1. Translation dislocations	96
3.2.2. Rotation dislocations	96
3.3. Dislocations in crystals	98
3.3.1. Symmetry, energy and mobility of perfect dislocations	98
3.3.2. Crystal plasticity and dislocation slip	100
3.3.3. Lattice friction to dislocation slip	104
3.3.4. Possible core splitting	111
3.3.5. Applications to different types of crystals	113
3.3.6. Highly mobile dislocations. Dislocation vibrations	125
3.3.7. Dislocation climb	128
3.3.8. Plastic deformation of amorphous solids	129
4. Conclusions	131
<i>Course 2. Defects in amorphous metals, by Frans Spaepen</i>	133
1. Introduction	136
2. Structural order and disorder	137
2.1. Structural order	137
2.1.1. Short-range order	137
2.1.2. Long-range order	141
2.2. Structural disorder	143
2.2.1. Definition of defects	143
2.2.2. Modeling of defects from crystalline analogues	143
2.2.3. Modeling of defects as structural fluctuations	145
2.2.4. Classification of defects according to transport function	146

3. Atomic Transport Properties	150
3.1. Isoconfigurational and equilibrium properties	150
3.2. Homogeneous plastic flow	152
3.2.1. Review of the experiments	152
3.2.2. Elementary theory of plastic flow	153
3.2.3. Discussion of the flow defect	155
3.3. Diffusion	156
3.3.1. Review of the experiments	156
3.3.2. Scaling of diffusivity and viscosity	158
3.3.3. Discussion of the diffusion defect	160
3.4. Conclusion	162
4. Deformation at high stresses	162
4.1. Deformation mechanism map	162
4.2. Inhomogeneous plastic flow	164
4.2.1. Phenomenology	164
4.2.2. Microscopic description	165
4.3. Fracture	169
4.3.1. Ductile fracture	169
4.3.2. Brittle fracture	171
References	172
<i>Seminar. The study of dislocations by electron microscopy, by Alain Bourret</i>	175
<i>Seminar. Defects in three- and two-dimensional colloidal crystals, by Paweł Pieranski</i>	183
<i>Seminar. Macroscopic random media: “Miam”, by Etienne Guyon</i>	201
<i>Course 3. Continuum theory of defects, by Ekkehart Kröner</i>	215
1. Introduction	219
2. Physical foundations of the defect field theory	222
2.1. The concept of defect and defect-free matter	222
2.2. Simplification of the general problem. Born–Oppenheimer approximation	223
2.3. Lattice versus continuum theory	225
3. The linear continuum theory of dislocations. Geometric-topological part	228
3.1. Continua with lattice structure	228

3.2. Elastic and plastic distortion	229
3.3. Compatibility and incompatibility. Qualitative description	235
3.4. First order differential form of the incompatibility law	238
3.5. Integral form of the incompatibility law	240
3.6. Second order differential form of the incompatibility law	242
3.7. Nye's lattice curvature (contortion). The tensor of incompatibility	243
3.8. Crystal dislocations versus continuum dislocations	247
3.9. Applications of Nye's equations to grain and phase boundaries	249
3.10. Summary of basic geometric laws of dislocation theory	253
 4. The linear continuum theory of dislocations. Static part	254
4.1. Incompatibility as the fundamental source of internal stresses.	
Quasidislocations	254
4.2. The basic equations of the internal stress problem	256
4.3. The second order stress function tensor. Isotropic medium	258
4.4. General and special solutions of the internal stress problem. Isotropic case	259
4.4.1. The general solution	259
4.4.2. Single dislocations	259
4.4.3. Plane problems	260
4.5. General and special solutions of the internal stress problem. Anisotropic case	262
4.5.1. The anisotropic elastic Green function	262
4.5.2. The anisotropic stress function ansatz	265
4.5.3. Special solutions of the anisotropic internal stress problem	270
4.6. The elastic energy of dislocation distributions	271
 5. Linear theory of point defects. Dia- and paraelasticity	272
5.1. Extrinsic and intrinsic point defects	272
5.2. The extrinsic elastic dipole and the extrinsic elastic polarizability	274
5.3. The intrinsic elastic dipole	280
5.4. The displacement field of the elastic dipole	282
5.5. Stress at a point. Amorphous media	283
 6. Differential geometry of defects in crystal lattices	284
6.1. The vacuum states of crystal lattices and their description by Euclidean geometry	284
6.2. Parallel displacement: the first basic law	289
6.3. Dislocation and the torsion of Cartan: the first basic identification	292
6.4. Length measurements in dislocated crystals. Uncertainty principle	294
6.5. Intuitive interpretation of the affine connection	296
6.6. Curvature tensor: the second basic law. Anholonomic coordinates	297
6.7. Point defects and Riemannian curvature: the second and third basic identification	300
6.8. Dislocations plus extrinsic point defects. Einstein tensor	304
6.9. Completion of the theory. Stress space and strain space	306
 7. Concluding remarks	307
References	312

<i>Course 4. Gauge theories and densities of topological singularities, by Igor Dzyaloshinskii</i>	<i>317</i>
1. Introduction	320
2. Conservation laws and exterior calculus	324
3. Topologically stable singularities in magnets and nematics	330
3.1. XY-model of a ferromagnet and a two-sublattice antiferromagnet	330
3.2. Heisenberg ferromagnets	331
3.3. Nematics	332
3.4. Multisublattice antiferromagnets	334
4. Gauge invariance and Yang–Mills fields	336
4.1. Maxwell electrodynamics	336
4.2. Yang–Mills fields and connections	338
4.3. Dirac monopole	342
5. Topological singularities and Yang–Mills fields	346
5.1. Multisublattice magnets	346
5.2. Ferromagnets and nematics	350
Appendix 1. Exterior calculus	354
Appendix 2. Derivation of (3.16)	358
General references	359
<i>Course 5. Classification topologique des défauts et des configurations des milieux ordonnés, par Louis Michel</i>	<i>361</i>
1. Introduction	364
2. Les groupes d'homotopie	366
2.1. Exemple 1: $X = S_1$	366
2.2. Exemple 2: $X = R^2 - \{0\}$	367
2.3. Digression	367
2.4. Exemple 3 et digression: $X = R^2 - \{a, b\}$	368
2.5. Autre famille d'exemples	369
2.6. Produit topologique; espaces contractibles	369
2.7. Le groupe d'homotopie du groupe des rotations. Son recouvrement universel	370
3. Quelques concepts fondamentaux au sujet des groupes	372
3.1. Sous-groupes invariants, groupes quotients	372
3.2. Le groupe d'homotopie $\pi_0$	374
3.3. Action de groupe, orbite, groupe d'isotropie	374
3.4. Orbites du groupe des rotations $S0(3)$ et de son recouvrement $SU(2)$	375
3.5. Homotopie des orbites de $S0(3)$	377

4. Etude topologique des défauts	378
4.1. L'orbite des états	378
4.2. La classification homotopique des défauts	378
4.3. Application aux champs (continus) de vecteurs	379
4.4. Les défauts des nématiques	380
5. Perspectives et conclusion	381
5.1. Succès de la classification homotopique	381
5.2. Puissance de la classification homotopique	382
5.3. Limitation de la classification homotopique	382
Références	383

*Course 6. Mathematical concepts in the theory of ordered media,  
by René Thom* 385

0. General introduction	388
1. Fundamentals of differential calculus (modern style)	388
1.1. The motivating problem	389
1.2. Smooth maps; local study	390
1.3. Tangent vectors and covectors: functional properties	391
1.4. Differential of a function. Invariance properties	394
2. Manifolds and transversality	395
2.1. Transversal maps. Regular points	395
2.2. Sard's theorem	397
2.3. Manifolds and bundles	397
2.4. Normal bundle to an embedded manifold	399
2.5. Transversality	400
3. Stratified spaces	401
3.1. Notions on analytical spaces	401
3.2. Regular point of an analytical space	402
3.3. Transversality on a stratified set. Isotopy lemma	402
3.4. Transversality in jet spaces	405
3.5. Examples. Real valued functions	406
3.6. Degenerate critical points of functions. Unfoldings	406
4. Phase diagrams due to Maxwell's convention	407
4.1. Maxwell's convention, Maxwell's set	407
4.2. Stratification of Maxwell's set	408
4.3. Application to phase diagrams. Gibbs phase rule	412
4.4. Phase transitions and Maxwell's rule	413
4.5. Other applications of Maxwell's rule and set (economy, geology etc.)	413
5. Ordered media and their defects	415
5.1. Groups and pseudo-groups	415
5.2. Definition of an ordered structure	415

5.3. Local and semi-local structures	416
5.4. Relative stability of ordered phases	418
5.5. Defects. Definition	418
5.6. The Kléman–Toulouse principle	418
5.7. Problems about the topological structure of defects	419
References	420
 <i>Seminar. Disentanglement of line defects in ordered media, by K. Jänich and Hans-Rainer Trebin</i>	421
 <i>Seminar. Many defect structures, stochasticity and incommensurability, by Serge Aubry</i>	431
 <i>Course 7. Singularities in waves and rays, by Michael Berry</i>	453
1. Introduction	456
2. Wavefront dislocations	457
2.1. Singularities of phase	457
2.2. Dislocation morphologies	460
2.3. Dislocations in acoustics, electromagnetism, quantum mechanics and water waves	467
3. Caustics as catastrophes	479
3.1. Singularities of ray families	479
3.2. Classification of stable caustics	488
3.3. Optical examples of the simplest catastrophes	492
3.4. Umbilic points: different classifications of the same singularity	501
3.5. Caustic networks	507
4. Diffraction catastrophes	517
4.1. Integral representations in shortwave asymptotics	517
4.2. Classification of diffraction patterns near stable caustics	527
4.3. Architecture of the simplest diffraction catastrophes	529
4.4. Projection identities	533
4.5. Scaling laws for diffraction catastrophes	538
5. Conclusion	540
References	541
 <i>Seminar. Dislocations and disclinations in transverse electromagnetic waves, by John Nye</i>	545

<i>Course 8. Structure and events in flow fields, by John Nye</i>	551
1. General approach	554
2. Two-dimensional steady flow	554
3. Events in time-dependent, two-dimensional flows	558
4. Events in time-dependent, three-dimensional flows	560
5. Review	562
Appendix A. The man in the boat	563
Appendix B. Generalized umbilic point on a glacier	564
References	568
<i>Course 9. Dislocations and earthquakes, by Raúl Madariaga</i>	569
1. Introduction	572
2. Earthquakes and plate tectonics	573
3. Earthquake related deformation	576
4. Fundamentals of seismic source radiation	579
4.1. Seismic moment tensors	583
4.2. Point moment tensors	584
4.3. Moment tensor equivalent to a dislocation loop	587
4.4. Eigenvalues of the moment tensor	589
5. The double couple model and seismic observations	590
6. Finite source models	595
6.1. Body wave radiation from a plane fault with arbitrary slip	597
6.2. Some general properties of far-field radiation	599
7. Dislocation models of faulting	600
8. Dynamic fault models	606
8.1. The static circular shear crack model	608
8.2. The dynamic circular shear crack model	610
References	614
<i>Seminar. From continental drift to dislocation cores, by Jean-Paul Poirier</i>	617

<i>Course 10. Dynamics of walls, lines and points in magnetic bubble garnets, by John C. Slonczewski</i>	631
1. Introduction	634
2. Wall dynamics in one dimension	635
3. Wall dynamics in three dimensions	643
4. Bubble as particle	645
5. Dynamics of Bloch lines	650
6. Bubble overshoot	654
7. Bloch points	657
8. Bubble resonance	660
References	662
<i>Course 11. Geometry and topology of defects in liquid crystals, bibliographical notes, by Yves Bouligand</i>	665
1. Introduction	668
2. Mesogenic molecules and structure of mesophases	668
3. Preparation and examination of defects	675
4. Resolution and contrast of defects	677
5. Conventions for the representation of molecular orientations in liquid crystals	678
6. Distortions in liquid crystals	681
7. Arrangement of layers around defects in smectic, cholesteric and columnar liquids	684
8. Topology of director fields	693
8.1. Nematics	693
8.2. Slightly twisted nematics	694
8.3. Smectics C	694
8.4. Twisted smectics C	696
8.5. Homotopy groups and classification of defects	697
9. Textures and defect associations	707
References	709

<i>Course 12. Amphiphilic mesophases made of defects, by Wolfgang Helfrich</i>	713
1. Introduction	716
2. Amphiphilic structures and their energies	718
2.1. Non-elastic theories	718
2.2. Curvature elasticity of fluid layers	720
2.3. Defects of bilayers and monolayer cylinders	725
3. Interpretation of amphiphilic mesophases	731
3.1. Mesophases between the lamellar phase (D) and the hexagonal phases (E and F)	731
3.2. Mesophase between the normal micellar solution ( $L_1$ ) and the normal hexagonal phase (E)	738
3.3. Satellites of the lamellar phase	739
3.4. Amphiphilic nematic liquid crystals	745
3.5. Remarks on microemulsions	748
4. Conclusion	753
References	754
<i>Seminar. Optical techniques for the analysis of defects in smectic liquid crystals, by Sven T. Lagerwall and B. Stebler</i>	757
<i>Course 13. Defects in ordered biological materials, bibliographical notes, by Yves Bouligand</i>	777
1. Introduction	780
2. Cellular and extracellular structures	780
2.1. The cell and its nucleus	780
2.2. Convecting motions in cytoplasm	784
2.3. Cytoplasmic organelles	785
2.4. Several cell systems	785
3. Defects in biological systems: general concepts	785
3.1. Order defects and organization defects	785
3.2. Orthotaxy, plethotaxy and cosmotaxy	787
3.3. Defects and teratology	788
4. One-, two- and three-dimensional arrays in cells and tissues	789
4.1. One-dimensional systems	789
4.2. Two-dimensional lattices	789
4.3. Three-dimensional lattices	793

5. Analogues of liquid crystals	793
5.1. Analogues of canonic liquid crystals	793
5.2. Analogues of smectic liquid crystals	796
5.3. Analogues of nematic liquid crystals	798
5.4. Analogues of cholesteric liquid crystals	800
6. Defects considered as natural experiments and possible morphogenetic factors	805
References	808

*Course 14. Statistical mechanics of topological defects,  
by Bertrand I. Halperin*

813

0. Preface	816
1. Introduction	816
1.1. Outline	817
2. Long-range, short-range and quasi-long-range order	819
3. The superfluid or planar-spin model	820
3.1. The Kosterlitz–Thouless theory	820
3.2. Dynamics of a superfluid film	822
3.3. Applications to superconducting films	824
3.4. Thick helium films	825
3.5. Effect of a symmetry-breaking perturbation	826
3.6. Comment on the 2-D Heisenberg model	828
4. Melting in two dimensions	829
4.1. Regular triangular lattice	829
4.2. “Experimental situation”	832
4.3. Effects of a crystalline substrate	833
4.4. Systems of tilted molecules	834
5. Defect mediated phase transitions in $d = 3$	837
5.1. Are defects necessary for the transition?	838
5.2. Are vortices sufficient in the 3-D XY-model?	840
5.3. Approximate solutions of the dislocation model of melting	841
6. Topological-charge correlation functions for the zeroes of a Gaussian vector field	844
6.1. Introduction	844
6.2. Definition of the Gaussian field	846
6.3. Results	848
6.4. Discussion	850
References	853