

Contents

Preface	xiii
Preface to the English Edition	xvii
Notation	xix
Chapter 1. Introduction to Phenomenological Phase Transition Theory	1
1. Fundamentals of Landau's Thermodynamic Theory	1
Elementary thermodynamic analysis (1). Spontaneous symmetry breaking at a continuous phase transition (7). The Landau condition for a second-order phase transition (9). Further development of Landau's theory (10).	
2. Prerequisites on Space-group Representations	12
Space-group irreducible representations (12). Irreducible representations and their decomposition (15).	
References	17
Chapter 2. Physical Realization of the Order Parameters at a Microscopic Level of Description	19
3. Tensor Representation of the Space Group on a Basis of Localized Atomic Functions	19
Constructing crystal space group reducible representations (20). The stabilizer method (24). Constructing basis functions for star arms (27).	
4. Permutational Representation and its Basis	28
A summary of formulas (28). The OP for ordering in AB type alloys (29). The OP for ordering in Nb-H and Ta-H hydrides (30).	

5. Vector Representation and its Basis	36
A summary of formulas (36). The OP at a structural phase transition in A-15 compounds (38). The OP at a structural phase transition in C-15 compounds (43).	
6. Pseudovector Representation and its Basis	45
A description of the magnetically ordered state (45). The OP at a magnetic phase transition in a garnet (47).	
References	51
Chapter 3. Symmetry Change at Phase Transitions	52
7. Change in Translational Symmetry	52
The Brillouin zone and the symmetric points in it (52). Arm mixing and the transition channel (56). Magnetic lattices (72).	
8. The Total Symmetry Change	74
Principles for finding the symmetry group of a new phase (74). An example of a group-theoretic method of searching for dissymmetric phases (77).	
9. Domains	82
Domains as a consequence of the Curie principle (82). A symmetry classification of domains (83). Arm, orientational and antiphase domains (85). Examples of an analysis of the domain structure (87).	
10. The Paraphase	91
The initial phase and the paraphase (91). Major criteria for paraphase search (93). An example of choosing the paraphase (94).	
References	98
Chapter 4. Analysis of the Thermodynamic Potential	100
11. Invariant Expressions of the Thermodynamic Potential	100
A straightforward (direct) method of constructing polynomial invariants (100). Constructing the Φ for the structural phase transition in C-15 compounds (101). Constructing the Φ for the structural phase transition in A-15 compounds (102).	
12. Integral Rational Basis of Invariants	104
General remarks (104). The IRBI construction algorithm (105). Solvability of the group and the minimal IRBI (110).	
13. Examples of the Construction of an IRBI	112
Constructing the IRBI for the structural transition in C-15 compounds (112). Construction of the IRBI for the structural transition	

CONTENTS

vii

in A-15 compounds (114). Constructing the IRBI for the structural transition in MnAs (117).	
14. Irreducible Representation Images and Thermodynamic Potential Types	118
General information on the I groups (118). Two- and three-component order parameters (119). A multicomponent order parameter (121). I groups and rotation groups in multidimensional space (121). Universal classes (123).	
References	130
Chapter 5. Phase Diagrams in the Space of Thermodynamic Potential Parameters 131	
15. Theoretical Fundamentals of the Phase Diagram Construction	
Method	131
Major physical principles (131). Requisite theorems from the algebra of polynomials (134).	
16. The One-Component Order Parameter	137
The form of the thermodynamic potential (137). The η^6 model (137). The η^8 model (139). Succession of solutions to equations of state (146).	
17. The Two-Component Order Parameter	148
The η^4 model (148). The η^6 model (152). Cubic invariants in the η^4 model (156). Cubic invariants in the η^6 model (159).	
18. The Three-Component Order Parameter	161
The phases and the stability conditions (161). The phase diagram in the η^4 model (164).	
19. The Role of the IRBI in the Construction of Phase Diagrams	165
The two-component order parameter (166). The three-component order parameter (167).	
20. Coupling Order Parameters	168
The interplay of two one-component order parameters (168). Phase transitions in MnAs (170). Phase transitions in KMnF ₃ (173). Orientation transitions (178).	
References	186

Chapter 6. Macroscopic Order Parameters	188
21. Transformation Properties of the Order Parameters	188
Physical realization of the macroparameters (188). Construction of basis functions (191). Constructing the thermodynamic potential (192).	
22. Interplay of Micro- and Macroparameters	197
Constructing the thermodynamic potential (197). Improper transitions (198). Examples of structural transitions in perovskite-type crystals (199).	
23. Ferroics	202
Classification of dissymmetric phases according to macroproperties (202). Ferroelectrics (204). Ferroelastics (205). Ferrobielectrics and ferrobimagnetics (205). Higher-order ferroics (206).	
24. Non-ferroics	211
Crystal-class-preserving phase transitions (211). Examples of non-ferroics (212).	
References	214
Chapter 7. Phase Transitions in an External Field	215
25. Phase Diagrams	215
Constructing a potential for a system in an external field (215). Phase diagram for the η^4 model (216). Phase diagram for the η^6 model (217). Singular points on the phase diagram (222). Multi-component order parameter (222). Splitting of a phase transition described by a microparameter in an external field (226).	
26. Features Peculiar to the Temperature Behavior of Susceptibility in the Vicinity of the Second-Order Phase Transition	227
Classification of the singularities by the Aizu indices (227). Catastrophe indices (231).	
27. Calculation of Susceptibilities for Second-Order Phase Transitions	233
Proper phase transitions (233). Improper phase transitions (234). Pseudoproper phase transitions (238).	
28. Singularities of the Susceptibility in the Vicinity of the First-Order Phase Transition	239
Classification of the first-order transitions (239). Classifying the singularities of susceptibilities (240). Calculating the susceptibilities (243).	

CONTENTS

ix

29. Domains in an External Field	244
A thermodynamic description of the domains (244). Effect of an external field on domains (247).	
References	249
Chapter 8. Martensite Transformations	250
30. Reconstructive Structural Transitions	250
Transitions without group-subgroup relation (250). Geometric relation of direct lattices (253). The b.c.c.-f.c.c. transition (255). The b.c.c.-h.c.p. transition (256). The f.c.c.-h.c.p. transition (257). Orientation relations (258). Interrelationship of reciprocal lattices (261).	
31. Thermodynamic Analysis of the Homogeneous State	266
Describing the martensitic transition in terms of deformation (266). Thermodynamic potential and phase diagram (268). Behavior in an external field (270). Shape memory effect (274).	
32. Inhomogeneous States in the Vicinity of the Phase Transition	275
Thermodynamic potential with gradient terms (275). Equations of motion (277). Tetragonal-strain phase transition (278). Phase transition with shear strain (282). Square lattice (288).	
33. The Omega Phase	292
Interrelationship between the lattices at the b.c.c.- ω -phase transformation (292). Thermodynamic potential (295). Inhomogeneous states (296).	
References	298
Chapter 9. Incommensurate Periodicity Phases	301
34. General Approach to the Problem	301
Commensurate and incommensurate phases (301). Expansion of a thermodynamic potential with continuous order parameters (303).	
35. Phases without Linear Gradient Terms in Free Energy	304
One-component order parameter (304). Two-component order parameter (309). Three-component order parameter (311). Lifschitz point (316).	
36. Phases with Linear Gradient Terms	318
The soliton lattice (318). Soliton lattice stability (323). The devil's staircase (328). Stochastic regime (331).	

CONTENTS

37. Multi-κ-structures	332
Conditions for many-arm structures to be thermodynamically favorable (332). Multi- κ -structure in CeAl ₂ (334). Multi- κ -structure in Nd (336).	
38. Incommensurate Phases in External Fields	340
Helical structure in an external field (340). Effect of external fields on the wave-vector of incommensurate phases (343).	
39. The Thermodynamics of Phase Transitions to Incommensurate Phases	347
Constructing a thermodynamic potential for non-Lifschitz stars (347). Phase transition from incommensurate to commensurate phase (350). Concomitant order parameters and incommensurate-phase symmetry (351). Peculiarities of susceptibilities in incommensurate phases (353).	
References	355
 Chapter 10. Color Symmetry and its Role in Phase Transition Theory 358	
40. Color Symmetry in the Theory of Magnetic Structures	358
Magnetic structures and their symmetries. Geometric aspect (358). Color symmetry and the thermodynamic-potential model (360). The hierarchy of approximations for describing the magnetic structure of FeGe ₂ (362).	
41. Supersymmetry of Incommensurate Structures	365
Incommensurate structures and the paradox of the Cheshire cat (365). Phase symmetry of the thermodynamic potential and the symmetry of incommensurate structures (367).	
42. Icosahedral Symmetry of Crystals. Quasicrystals	372
A new type of atomic ordering (372). The geometric aspect (374). The thermodynamic aspect (379). The symmetry of quasicrystal structures (385). Al ₈₆ Mn ₁₄ : A Fibonacci structure or a quasicrystal? (387). Conclusion (388).	
43. Color Groups in the Theory of Systems with a Quantum Mechanical Order Parameter	390
The quantum mechanical order parameter (390). Classification of ordered phases on the basis of color sub-groups of the normal-phase	

CONTENTS

xi

symmetry group (391). OP symmetry and the behavior of the gap in κ space (394). Thermodynamic potentials (399).	
References	401
Chapter 11. Fluctuations and Symmetry	404
44. Fundamentals of the Fluctuation Phase Transitions Theory	404
Critical indices (404). The renormalization-group and ϵ -expansion method (406). The isotropic model (413).	
45. Critical Behavior of Anisotropic Systems	415
Universal classes (415). Cubic anisotropy (417). Examples of systems with multicomponent order parameters (419).	
46. Fluctuation-Induced Break-Down to First-Order Phase Transitions .	423
The absence of stable fixed points (423). First-order transitions in magnetic systems (424).	
47. Fluctuations in the Vicinity of Multicritical Points	426
Systems with coupled order parameters. Bicritical and tetracritical points (426). Lifschitz point (434).	
References	437
Appendix	439
Index	441