



CONTENTS

PART I. GRAVITATION IN LOCALIZED SYSTEMS

An Introduction to the Theory of Gravitational Radiation . . . . .	3
Thibault Damour	
1. Introduction	
1.1 Scope of these lectures	
1.2 Conventions and Notation	
2. What is a Gravitational Wave?	
2.1 First viewpoint: propagation of discontinuities	
2.2 Second viewpoint: high-frequency waves	
2.3 Third viewpoint: weak gravitational waves on a flat background	
2.4 Other viewpoints	
3. Basic Problems of Gravitational Radiation Theory	
3.1 A catalogue of problems	
3.2 A catalogue of approximation methods	
4. Quadrupole Moment Formalisms	
4.1 “Quadrupole laws” versus “quadrupole equations”	
4.2 The three “quadrupole laws”	
4.3 General discussion of “quadrupole equations”	
4.4 The “standard,” or “Einstein,” far-field quadrupole equation	
4.5 The “Landau-Lifshitz” quadrupole equation	
4.6 The “Fock” quadrupole equation	
4.7 Recent improvements	
4.8 Thorne’s generation formalism	
4.9 Tentative conclusion about the “standard” far-field quadrupole equation	
4.10 Far-field quadrupole equations for some specific problems	
4.11 The generalized quadrupole equation of Halpern-Desbrandes and Press	
4.12 The far-field multipole law	
4.13 On the definition of the asymptotic outgoing radiation field	
4.14 Energy-loss quadrupole equations	
4.15 Radiation-reaction quadrupole equations à la Burke-Thorne	
4.16 Post-Newtonian radiation-reaction quadrupole equations	
4.17 Radiation-reaction quadrupole equations in the N-body problem	
4.18 Conclusion	
5. Multipolar-Post-Minkowskian Formalisms	
5.1 Introduction	
5.2 Formal framework	
5.3 The hierarchy of equations to be solved	
5.4 The first step of the hierarchy	

- 5.5 On the meaning of the “algorithmic multipole moments”
- 5.6 Recursive algorithm for constructing the higher steps of the hierarchy
- 5.7 Partial results on the asymptotic problem
- 5.8 Preliminary results on the propagation problem
- 5.9 Partial results on the generation problem
- 5.10 Preliminary results on the radiation-reaction problem
- 6. Gravitational Radiation and Binary Systems of Condensed Objects
  - 6.1 One method for two questions
  - 6.2 The two-condensed-body problem in General Relativity
  - 6.3 The internal problems
  - 6.4 The matching: internal  $\rightarrow$  external
  - 6.5 The external problem
  - 6.6 A convenient auxiliary mathematical technique
  - 6.7 Answer to the first question of §6.1 (generation)
  - 6.8 Equations of motion of a binary system of condensed bodies
  - 6.9 Radiation-reaction force versus the relativistic Laplace effect
  - 6.10 Poincaré on gravitational waves
  - 6.11 Answer to the second question of §6.1 (radiation reaction)
  - 6.12 Application to the binary pulsar PSR 1913+16
  - 6.13 Conclusion
- Acknowledgements
- Some Books Fully Devoted to Gravitational Radiation
- Bibliographical References

## Mathematical Foundations of the Theory of Relativistic Stellar and Black

Hole Configurations . . . . .	63
Brandon Carter	

- 1. Introduction
  - 1.1 Background
  - 1.2 Purpose and Plan
- 2. Notions of General (Dynamic) Black Hole Theory
  - 2.1 Definition
  - 2.2 Kinematics of Characteristic (Null) Boundaries
  - 2.3 Generalised Raychaudhuri Equation for Timelike and Null Geodesic Congruences
  - 2.4 The Horizon of a Black Hole
  - 2.5 Asymptotic Predictability, Closed Trapped Surfaces, and Apparent Horizons
  - 2.6 Cosmic Censorship and the Existence of an Asymptotic Equilibrium
  - 2.7 Approximate Equilibrium
- 3. Stationary and Static Equilibrium
  - 3.1 Overview
  - 3.2 Elementary Local Properties of Killing Horizons
  - 3.3 Uniformity of the Corotating Potential on a Killing Horizon
  - 3.4 Uniformity of  $\kappa$  (the “zeroth law”) on a Killing Horizon
  - 3.5 The Mass of a Stationary System
  - 3.6 Globally Bradyonic Character of Generators of Stationary D.O.C.
  - 3.7 The Staticity Theorem for Non-Rotating Electromagnetic Black Holes
  - 3.8 Local Characterisation of Simply Connected Static Domain of Outer Communications
- 4. Axisymmetric Equilibrium States
  - 4.1 Mass and Angular Momentum of Stationary Axisymmetric Systems
  - 4.2 Circularity Theorem for Stationary Axisymmetric Systems
  - 4.3 The Ergoregion and ZAMOs in Papapetrou Coordinates

4.4	Slowly Rotating (but Strongly Gravitating) Stellar Equilibrium Configurations	
4.5	Killing Horizon (Nullity and Rigidity) Property of Locus Where ZAMOs Go Null	
4.6	Black Hole Mass and Angular Momentum and their Variations	
4.7	Superradiance	
4.8	Local Characterisation of Simply Connected Stationary (Circularity) Axisymmetric D.O.C.	
5.	The Source Free Equilibrium State Problem for Axisymmetric Black Holes	
5.1	Canonical Global Coordinate System for the D.O.C. of a Stationary Axisymmetric Black Hole	
5.2	Reduction to a 2-dimensional Boundary Problem	
5.3	The Final Step in the Uniqueness Theorem	
5.4	Killing-Maxwell-Yano System	
5.5	The Canonical Tetrad	
	Acknowledgements	
	References	
Relativistic Gravitational Instabilities	. . . . .	123
	Bernard F. Schutz	
	Introduction	
	Spherical Pulsation of Spherical Stars	
	Newtonian stars	
	Relativistic stars	
	The turning point criterion for white dwarfs and neutron stars	
	Star clusters	
	Nonspherical Pulsation of Spherical Stars	
	Newtonian stars	
	Relativistic stars	
	Strongly damped modes	
	Quadrupole gravitational radiation	
	Nonspherical Perturbations of Spherical Black Holes	
	Formulation as a scattering problem	
	Calculations of the normal modes	
	Stability of Rotating Stars: General Remarks	
	The Maclaurin Spheroids	
	The nonaxisymmetric modes	
	The secular instabilities	
	The T/W criterion for instability	
	A Relativistic Approach to Stability	
	Perfect fluids in general relativity	
	Definition of a perturbation in terms of a sequence of solutions	
	Two preferred perturbations; Eulerian and Lagrangian	
	Perturbations of Einstein's Equations	
	A stability criterion	
	A Simple Approach to the Radiation Instability	
	Conserved quantities for wave fields	
	Mechanism for the gravitational wave instability	
	Gravitational wave instability as a two-stream instability	
	Other ways of exciting the instability	
	The instability due to viscosity	
	The Perturbed Energy of a Rotating System	
	Orbiting particle: an elementary example	
	The second-order energy of a rotating fluid	

Accretion and Collapse . . . . .	155
D. Lynden-Bell	
I. The Gravothermal Catastrophe	
1. Specific Heats	
2. A Thought Experiment	
3. Why Self-Similar Solutions Occur in Science	
4. Evolution After Core Collapse	
II. Spherical Accretion	
5. Bondi Accretion	
6. Relativistic Accretion	
7. Cold Self-Similar Gravitational Collapse	
III. Disk Accretion	
8. Energy, Angular Momentum and Dissipation	
9. Viscous Newtonian Accretion Disks	
10. Relativistic Accretion Disks	
IV. Optically Thick Accretion	
11. Self-Similar Solutions	
12. Enthalpy Theorem and Jet Production	
Accretion Disk Electrodynamics . . . . .	195
M. Kuperus	
1. The Standard Thin Disk	
2. Turbulent Dynamo in Accretion Disks	
3. Electrodynamic Coupling of Accretion-Disk Coronae	
4. The Interaction of a Neutron Star with an Accretion Disk	
4.1 Spin up of neutron stars	
4.2 Quasiperiodic oscillations	
References	

## SPECIAL TOPICS I

The Membrane Paradigm for Black-Hole Astrophysics . . . . .	209
Kip S. Thorne	
Tidal Disruption . . . . .	215
J.P. Luminet	
1. Introduction	
2. Tidal Tensor	
3. Ellipsoidal Deformations of Homogeneous Bodies	
3.1 The stationary rotational problem	
3.2 The stationary tidal problem	
3.3 The dynamical tidal problem	
4. Tidal Deformations of a Compressible Body	
4.1 The tidal rolling mill effect	
4.2 Tidal versus collisional disruption of stars	
4.3 The affine star model	
4.4 Motion in relativistic tidal field	
4.5 Pancake nucleosynthesis and the fate of debris	
References	

Naked Singularities in Spherical Gravitational Collapse . . . . .	229
D.M. Eardley	
1. Introduction	
2. Dust Collapse and Shell Focusing Singularities	
2.1 Shell-crossing	
2.2 Tolman-Bondi solutions	
2.3 Causal structure of the Tolman-Bondi solutions	
2.4 Interpretation of shell-focusing singularities	
2.5 Collapse of null fluid	
3. Collapse of Scalar Field Configurations	
3.1 Self-similar collapse	
3.2 Generic spherical collapse	
Acknowledgements	
References	

## II. GRAVITATION IN COSMOLOGY

Some Topics in Relativistic Cosmology . . . . .	239
John D. Barrow	
Introduction	
Orientation	
The Universe is Unique	
Non-local Influences	
Horizons	
How Many Spatial Dimensions are There?	
Variation of Fundamental "Constants"	
Unknown Physics	
Selection Effects	
Unknown Matter Fields	
How Little Could We Know?	
Newtonian Gravitation	
Newtonian Cosmology	
General Relativistic Cosmology	
The Friedman Cosmological Models	
Observable Parameters	
When Do Closed Universes Recollapse?	
Spatially Homogeneous Universes	
The Microwave Background and the Density of the Universe	
Microwave Background Observations	
Characteristic Microwave Background Patterns	
Quadrupole	
Hotspot	
Spirals	
Observational Limits	
Isotropy and Homogeneity	
The Cosmological Principle(s)	
Can We Prove a Cosmological Principle?	
Is Isotropy a Stable Property of Cosmological Models?	
Is Isotropy Really Unstable and Does it Matter Anyway?	
Approach to a Family of Plane Waves	
No Hair Theorems	
Inflation and the Initial Value Problem	
Inflation and the Strong Energy Condition	
The Deflationary Universe	
Resumé	
Acknowledgements	
References	

Cosmic Strings and the Origin of Structure in the Universe . . . . .	293
D.M. Eardley	
1. Introduction	
1.1 Origin of Perturbations in the Universe	
2. Quantum Particle Creation in an Inflationary Universe	
2.1 A Simple Model	
2.2 More Realistic Models	
3. Topological Defects in Field Theories	
3.1 Cosmological Constant	
3.2 Domain Walls	
3.3 Cosmic Strings	
3.4 Monopoles	
3.5 Instantons	
4. Cosmological Evolution of Topological Defects	
4.1 Bounds on Evolution of Defect Density	
4.2 Evolution of Monopoles and Domain Walls	
4.3 Evolution of Cosmic Strings	
5. Motion and Evolution of Cosmic Strings	
5.1 Closed Loops	
5.2 Gravitational Waves from Closed Loops	
5.3 Galaxy Formation by Cosmic Strings	
5.4 Fluctuations in the Cosmic Background Radiation Due to Cosmic Strings	
5.5 Light Bending Due to Cosmic Strings	
Acknowledgements	
References	
Cosmological Phase Transitions . . . . .	307
Edward W. Kolb	
1. The Evolution of the Vacuum	
1.1 High Temperature Symmetry Restoration	
1.2 Domain Walls	
1.3 Cosmic Strings	
1.4 Magnetic Monopoles	
1.5 The Kibble Mechanism	
2. Inflation	
2.1 Loose Ends of the Standard Cosmology	
2.2 Inflation – The Basic Picture	
2.3 Dynamics of Inflation	
2.4 Specific Models	
2.5 Present Status and Future Directions	
Acknowledgements	
References	
Prediction in Quantum Cosmology . . . . .	329
James B. Hartle	
1. Introduction	
2. Predictions from the Wave Function of the Universe	
2.1 The Wave Function of the Universe	
2.2 Cosmological Observations and Cosmological Predictions	
2.3 The Nature of Cosmological Predictions	
2.4 Quantum Mechanics of Individual Systems	
2.5 The Problem of Time	
3. Laws for Initial Conditions	

- 3.1 The Sum Over Histories Formulation of Quantum Cosmology
- 3.2 Constraints
- 3.3 A Proposal for a Wave Function of the Universe
- 4. The Limit of Classical Geometry and Quantum Field Theory in Curved Spacetime
  - 4.1 The Semiclassical Approximation to Non-Relativistic Particle Quantum Mechanics
  - 4.2 The Born-Oppenheimer Approximation for Real Clocks
  - 4.3 The Approximation of Quantum Field Theory in Curved Spacetime
  - 4.4 The Semiclassical Vacuum
- Acknowledgements
- Problems
- References

SPECIAL TOPICS II

The Quasi-Isotropic Universe . . . . .	361
D.J. Raine	
1. Introduction	
2. The Microwave Background	
3. Helium Abundance	
4. Anisotropic Spatially Homogeneous Cosmologies	
5. The Wainwright and Anderson Solution	
6. An Inhomogeneous Model	
7. Discussion of Results	
7.1 The Anthropic Principle	
7.2 Initial Conditions	
7.3 Gravitational Entropy	
References	
Semiclassical Quantum Gravity in Two and Four Dimensions . . . . .	371
N. Sánchez	
Introduction	
1. Quantum Effects Near Distorted Black Holes	
2. Q.F.T and the Antipodal Identification of Black Holes and of Desitter Space	
3. The Back-Reaction Problem in Two Dimensions: Liouville and Schroedinger Equations	
References	
Towards a Theory for the Quantum Mechanics of Gravitational Collapse . .	383
G. 't Hooft	
Abstract	
1. Introduction	
2. String Theory in a Nut Shell	
3. The Black Hole	
4. The Shifting Horizon	
5. A Link with String Theory	
6. Conclusion	
References	
Index . . . . .	397