



# Contents

<b>Introduction</b>	<b>I</b>
<b>PART I DYNAMICAL PRINCIPLES</b>	<b>13</b>
<b>Chapter 1 Principles of Analytical Dynamics</b>	<b>15</b>
VARIATIONAL PRINCIPLES	15
1.1 Hamilton's Principle	15
1.2 Relation to the Calculus of Variations	19
1.3 Canonical Equations	22
1.4 Covariance	29
1.5 Geometry of Motion	36
1.6 Equivalence of Dynamical Systems	40
CANONICAL TRANSFORMATIONS	42
1.7 Bilinear Covariants	42
1.8 Contact Transformations	47
1.9 Lagrange's and Poisson's Brackets	48
1.10 Canonical Transformations	51
1.11 Nonholonomic Constraint	59
THE HAMILTON-JACOBI PARTIAL DIFFERENTIAL EQUATION	61
1.12 The Hamilton-Jacobi Theorem	61
1.13 Hydrodynamical Proof of Jacobi's Theorem	65
1.14 Variation of Constants	67
1.15 Liouville's Theorem	70
1.16 Stäckel's Theorem	73
1.17 Condition for Separability	77

	HAMILTONIAN MATRIX	82
1.18	Hamiltonian Matrix	82
1.19	Canonical Transformation	85
1.20	Pfaff's Criterion	88
1.21	Polar Factorization	90
1.22	Pfaffian Dynamical System	94
<b>Chapter 2</b>	<b>Quasi-Periodic Motion</b>	<b>104</b>
	SIMPLY PERIODIC MOTION	104
2.1	Lebesgue's Integrals	104
2.2	Fourier Series	113
2.3	Lagrange's Theorem	117
2.4	Asymptotic Motion and Libration	119
	CONDITIONALLY PERIODIC MOTION	123
2.5	Multiply Periodic Functions	123
2.6	Weierstrass's Auxiliary Theorem	128
2.7	Conditionally Periodic Motion	132
2.8	Kronecker's Theorem	137
2.9	Motion of Molecules in a Cube	142
2.10	Quasi-Periodic Function of Bohl	144
	ALMOST PERIODIC FUNCTIONS	149
2.11	Definition	149
2.12	Properties of Almost Periodic Functions	151
2.13	Base Field	159
2.14	Quasi-Periodic Functions and Almost Periodic Functions	162
	RELATIVISTIC ONE-BODY PROBLEM	166
2.15	Transformation of the Problem	166
2.16	Relativistic Field of Schwarzschild	168
2.17	The Complete Integral	171
2.18	Solution by Elliptic Functions	174
2.19	Discussion of the Motion	178
2.20	Algebraic Integrals	185
	APPLICATION TO AN EARTH SATELLITE	196
2.21	Earth Potential	196
2.22	Oblate Spheroidal Coordinates	199
2.23	Integration by Elliptic Integrals	205
2.24	Reduction to the Two-Fixed-Center Problem	215
<b>Chapter 3</b>	<b>Particular Solutions of the Many-Body Problem</b>	<b>228</b>
	HOMOGRAPHIC MOTIONS	228
3.1	Fundamental Equations	228

---

	STRAIGHT-LINE HOMOGRAPHIC MOTIONS	232
3.2	Straight-Line Configuration	232
	PLANE HOMOGRAPHIC MOTIONS	237
3.3	Condition for Plane Configurations	237
3.4	Central Figures	242
3.5	Properties of Central Figures	246
	SPACE HOMOGRAPHIC MOTIONS	256
3.6	Condition for Space Configurations	256
	ISOSCELES-TRIANGULAR SOLUTIONS	259
3.7	Regular and Irregular Singularities	259
3.8	Existence of the Solutions	268
3.9	Nature of the Isosceles-Triangular Solutions	276
<b>PART II TRANSFORMATION THEORY</b>		
<b>Chapter 4</b>	<b>Continuous Groups of Transformations</b>	291
	INFINITESIMAL TRANSFORMATIONS	291
4.1	The First Integrals	291
4.2	Infinitesimal Transformations	296
4.3	Poisson's Theorem	301
	LIE'S THEOREM	303
4.4	Elimination of Time	303
4.5	Involution	304
4.6	Invariantive Relations	305
4.7	Liouville's Theorem	310
4.8	Lie's Theorem	312
	FUNCTION GROUPS	316
4.9	Complete Systems	316
4.10	Function Groups	320
4.11	Canonical Form of a Function Group	323
	INTEGRALS OF THE $n$ -BODY PROBLEM	328
4.12	Eulerian Integrals	328
4.13	Distinguished Functions	333
4.14	Parametrization	336
4.15	Separating Parametrization	338
4.16	First Integrals	342
	INTEGRAL INVARIANTS	343
4.17	Integrals and Integral Invariants	343
4.18	Condition for an Integral Invariant	346
4.19	Integral Invariants along Trajectories	350

4.20	Variational Equations	352
4.21	Integral Invariants of the $n$ -Body Problem	359
4.22	Cartan's Exterior Multiplication	363
	ADIABATIC INVARIANTS	381
4.23	Adiabatic Invariants	381
4.24	Adiabatic Approximation	391
4.25	Existence of Adiabatic Invariants	401
	LIE GROUPS	406
4.26	Lie's Fundamental Theorems	406
4.27	Adjoint Groups	419
4.28	Simple and Semisimple Groups	428
4.29	Representation of a Lie Group	436
<b>Chapter 5</b>	<b>Reduction of the <math>n</math>-Body Problem</b>	<b>447</b>
	TRANSFORMATION OF LAGRANGE	447
5.1	Transformation of Lagrange	447
5.2	Levi-Civita's Transformation	453
	TRANSFORMATION OF POINCARÉ	466
5.3	Transformation of Poincaré	466
5.4	Elimination of the Nodes	468
5.5	Transformation $\alpha$	470
5.6	Transformation $\beta$	472
5.7	Transformation $\gamma$	475
5.8	Reduction from $6n - 6$ to $6n - 10$	476
	ELLIPTIC MOTIONS	482
5.9	Motion in Conic Sections	482
5.10	Elliptic Motions	487
5.11	Cauchy's First Theorem	490
5.12	Cauchy's Second Theorem	496
5.13	Expansion in Several Variables	503
5.14	Radius of Convergence	510
5.15	Andoyer's Expansion	521
	VARIATION OF CONSTANTS	526
5.16	Delaunay's Elements	526
5.17	Poincaré's Elements	531
5.18	Keplerian Elements	537
5.19	Angular Momentum Integrals	540
5.20	Other Kinds of Canonical Elements	545
<b>Chapter 6</b>	<b>Algebraic Integrals and Uniform Integrals</b>	<b>556</b>
	THEOREM OF BRUNS	556
6.1	Integrals Independent of Time	556

6.2	Lemma I	559
6.3	Lemma II	563
6.4	Integrals of Momentum and Angular Momentum	568
6.5	Energy Integral	577
6.6	Integrals Containing Time	581
	PAINLEVÉ'S EXTENSION	584
6.7	Preparatory Lemmas	584
6.8	Properties of (S)	588
6.9	Properties of (S <sub>0</sub> )	590
6.10	Properties of the Integrals of (S)	596
6.11	Application to the $n$ -Body Problem	600
	INTEGRALS OF THE RESTRICTED THREE-BODY PROBLEM	604
6.12	Poincaré's Recurrence Theorem	604
6.13	Lemma	607
6.14	Proof of Siegel's Theorem	611
	THEOREM OF POINCARÉ	613
6.15	Case of Nonvanishing Hessian	613
6.16	Case of Vanishing Hessian	616
6.17	Application to the $n$ -Body Problem	624
6.18	Discussion of Poincaré's Theorem	629
6.19	Integrals without Parameters	632
	KOWALEWSKI'S TOP MOTION	639
6.20	Rigid-Body Rotation	639
6.21	Solution in Algebraic Integrals	642
6.22	Abel's Lemmas	648
6.23	Kowalewski's Solution	653
6.24	Riemann $\vartheta$ -Functions	661
6.25	Hyperelliptic Functions	671
	<b>Retrospect of Volume 1</b>	683
	<b>Index</b>	685