

Contents

Preface	v
Chapter 1	
General Considerations on the Euler Equation	1
1.1. The Equation of Motion of an Ideal Incompressible Fluid	1
1.2. Vorticity and Stream Function	9
1.3. Conservation Laws	23
1.4. Potential and Irrotational Flows	33
1.5. Comments	40
Appendix 1.1 (Liouville Theorem)	48
Appendix 1.2 (A Decomposition Theorem)	49
Appendix 1.3 (Kutta–Joukowski Theorem and Complex Potentials) .	50
Appendix 1.4 (d'Alembert Paradox)	52
Exercises	55
Chapter 2	
Construction of the Solutions	59
2.1. General Considerations	59
2.2. Lagrangian Representation of the Vorticity	63
2.3. Global Existence and Uniqueness in Two Dimensions	66
2.4. Regularity Properties and Classical Solutions	72
2.5. Local Existence and Uniqueness in Three Dimensions	74
2.6. Some Heuristic Considerations on the Three-Dimensional Motion	78
2.7. Comments	80
Appendix 2.1 (Integral Inequalities)	86
	ix

Appendix 2.2 (Some Useful Inequalities)	87
Appendix 2.3 (Quasi-Lipschitz Estimate)	89
Appendix 2.4 (Regularity Estimates)	89
Exercises	90
Chapter 3	
Stability of Stationary Solutions of the Euler Equation	93
3.1. A Short Review of the Stability Concept	93
3.2. Sufficient Conditions for the Stability of Stationary Solutions: The Arnold Theorems	104
3.3. Stability in the Presence of Symmetries	115
3.4. Instability	120
3.5. Comments	128
Exercises	132
Chapter 4	
The Vortex Model	134
4.1. Heuristic Introduction	134
4.2. Motion of Vortices in the Plane	137
4.3. The Vortex Motion in the Presence of Boundaries	152
4.4. A Rigorous Derivation of the Vortex Model	157
4.5. Three-Dimensional Models	166
4.6. Comments	169
Exercises	176
Chapter 5	
Approximation Methods	178
5.1. Introduction	178
5.2. Spectral Methods	179
5.3. Vortex Methods	182
5.4. Comments	186
Appendix 5.1 (On K-R Distance)	188
Exercises	189
Chapter 6	
Evolution of Discontinuities	191
6.1. Vortex Sheet	191
6.2. Existence and Behavior of the Solutions	200
6.3. Comments	207
6.4. Spatially Inhomogeneous Fluids	211
6.5. Water Waves	212
6.6. Approximations	219
Appendix 6.1 (Proof of a Theorem of the Cauchy-Kowalevski Type)	227
Appendix 6.2 (On Surface Tension)	228

Chapter 7	
Turbulence	230
7.1. Introduction	230
7.2. The Onset of Turbulence	234
7.3. Phenomenological Theories	246
7.4. Statistical Solutions and Invariant Measures	250
7.5. Statistical Mechanics of Vortex Systems	256
7.6. Three-Dimensional Models for Turbulence	268
References	272
Index	281