

CONTENTS

Foreword	9
Introduction	11
Chapter 1. Classical hydrodynamic stability	17
§ 1.1. Setting of the problem	17
1.1.1. Stability in the small	17
1.1.2. Stability in the mean	21
1.1.3. Linear stability	26
1.1.4. Rayleigh and Squire's theorems	37
1.1.5. Global stability	40
1.1.6. Stability of the mean motion in transition regime	42
§ 1.2. Orr-Sommerfeld equation	45
1.2.1. Nonviscous Orr-Sommerfeld equation	45
1.2.2. Tollmien's solution of the Blasius' boundary layer problem	47
1.2.3. Relationship between Tollmien's and Heisenberg's solutions	52
§ 1.3. Criteria of hydrodynamic stability	54
1.3.1. Serrin's universal criteria	55
1.3.2. Syngé's criterion for the Couette flow between rotating cylinders	58
1.3.3. Syngé's criterion for plane parallel flows	61
1.3.4. Joseph's theorems	64
1.3.5. The envelope method	71
<i>References</i>	73
Chapter 2. Generalized solutions in hydrodynamic stability	77
§ 2.1. Function spaces	77
2.1.1. Spaces of continuous functions	78
2.1.2. The L^p spaces	79
2.1.3. Generalized derivatives	82
2.1.4. Sobolev spaces	84
2.1.5. Embedding theorems	85
2.1.6. Compactness in the L^p spaces	88
2.1.7. Spaces of vector functions	89
2.1.8. Solenoidal vectors	90
2.1.9. Functions of time	92
§ 2.2. Types of solutions in hydrodynamic stability theory	93
2.2.1. Classical solutions	93
2.2.2. Generalized solutions of the linear problem	94
2.2.3. Generalized solutions of the nonlinear problem	99
2.2.4. Existence of generalized solutions	104

§ 2.3.	Completeness of normal modes	112
2.3.1.	Motions in bounded domains	112
2.3.2.	Motions in unbounded domains	115
§ 2.4.	Linearization principle	118
2.4.1.	The finite-dimensional case	118
2.4.2.	Linearization principle in hydrodynamic stability	120
2.4.3.	Stability of plane Couette flows	128
§ 2.5.	The principle of exchange of stabilities (P.E.S.)	135
2.5.1.	The neutral state and P.E.S.	135
2.5.2.	Proof of P.E.S. for particular motions	136
2.5.3.	Branching (bifurcation) of solutions of hydrodynamic equations	137
§ 2.6.	Universal criteria of hydrodynamic stability	140
2.6.1.	Stationary basic flows	140
2.6.2.	Nonstationary basic flows	143
<i>References</i>	147
Chapter 3.	Branching and stability of the solutions of the Navier-Stokes equations	152
§ 3.1.	Topological degree method for nonlinear equations in Banach spaces (Leray-Schauder method)	152
3.1.1.	The finite-dimensional case	152
3.1.2.	The infinite-dimensional case (Leray-Schauder method)	156
§ 3.2.	Branching of solutions of the Navier-Stokes equations by the Leray-Schauder method	161
3.2.1.	Convective motions	161
3.2.2.	Couette flow in the case of a fixed exterior cylinder	167
3.2.3.	Flows between two cylinders rotating in the same direction	170
3.2.4.	Flows in bounded domains	174
3.2.5.	Kolmogorov's flows	174
§ 3.3.	Liapunov-Schmidt method	176
3.3.1.	The case of integral equations	176
3.3.2.	The case of nonlinear equations in Banach spaces ..	180
§ 3.4.	Branching of solutions of the Navier-Stokes equations by the Liapunov-Schmidt method	186
3.4.1.	Convective motions	186
3.4.2.	Couette motion	191
3.4.3.	Motions in bounded domains	194
3.4.4.	The stability of branching solutions	196
§ 3.5.	Hopf bifurcation by the Joseph-Sattinger method	200
3.5.1.	Deduction of secondary solutions	200
3.5.2.	Stability of secondary solutions	205
§ 3.6.	Generation of turbulence by instability and local branching	207
<i>References</i>	209

Chapter 4. Nature of turbulence	212
§ 4.1. Leray model	212
§ 4.2. The Landau-Hopf conjecture	213
§ 4.3. The Ruelle-Takens theory	216
4.3.1. The case of the Navier-Stokes equations.....	216
4.3.2. The Lorenz model	219
§ 4.4. Generic finiteness of the set of the solutions of the Navier-Stokes equations	221
§ 4.5. Pattern formation; symmetry breaking instability....	225
§ 4.6. Concluding remarks; open problems	225
<i>References</i>	228
Chapter 5. The influence of the presence of a porous medium on hydrodynamic stability	231
§ 5.1. The mathematical problem	231
§ 5.2. Rayleigh-Taylor instability	234
§ 5.3. The Kelvin-Helmholtz instability	236
§ 5.4. The case of a vertical cylinder	243
<i>References</i>	246
Appendices	
1. Operators in Hilbert spaces	248
2. Semigroups of operators in Banach spaces.....	255
3. Spectral theory of linear operators	256
4. Calculus of variations	261
5. Geometric methods in branching theory.....	265
6. New methods for solving the Orr-Sommerfeld equation.....	269
7. Analytical methods to solve some eigenvalue problems in hydrodynamic and hydromagnetic stability theory.....	274
8. Stability of nonstationary fluid flows.....	297
<i>Afterword</i>	299
<i>Index</i>	305