Contents

CHAPTER 1

Vectors and Cartesian Tensors 3

1.1 Introduction 3
1.2 Index Notation of a Vector 4
1.3 Transformation Law of a Vector 6
1.4 Rules of the Index Notation and the Transformation Laws of Cartesian Tensors 7
1.5 Addition and Subtraction of Vectors and Cartesian Tensors Using the Index Notation 11
1.6 Scalar (or Dot) Product of Two Vectors 12
1.7 Vector (or Cross) Product of Two Vectors 13
1.8 Multiplication of Cartesian Tensors 15
1.9 Gradient of a Scalar Function of Position (or Scalar Field) 16
1.10 Divergence of a Vector Function of Position (or Vector Field) 17
1.11 Curl of a Vector Function of Position (or Vector Field) 19
1.12 The "ϵ-δ Identity" 19
1.13 The Line Integral and Stokes' Theorem 21
1.14 Conservative Vector Field and the Concept of a Scalar Potential Function 24
1.15 Gauss' Theorem and Its Generalization to Tensor Fields 25
1.16 Green's Theorem 27
Summary 28
Problems 30

CHAPTER 2

Analysis of Stress in a Continuum 34

2.1 The Continuum Approach 34
2.2 Definitions of Fundamental Properties 35

2.3 Classification of Continuous Media 36
2.4 Body Forces and Surfaces Forces 37
2.5 Components of Stress or Stress Tensor 38
2.6 Force and Moment Equations of Equilibrium 42
2.7 Transformation Law for the Components of Stress or Stress Tensor 46
2.8 The Stress Quadric 49
2.9 Principal Stresses and Principal Axes 50
2.10 The Stress Vector and Normal Stress Referred to the Principal Axes 54
2.11 Stress Invariants 55
2.12 The Stress Deviator Tensor 56
2.13 Maximum Shearing Stress 58
Summary 61
Problems 63

CHAPTER 3

Analysis of Deformation in a Continuum 67

3.1 Lagrangian and Eulerian Descriptions of Deformation or Flow 67
3.2 The Comoving Derivative 69
3.3 Velocity and Acceleration 70
3.4 The Continuity Equation 73
3.5 Strain Tensors 79
3.6 The Linear Rotation Tensor and Rotation Vector; Analysis of Relative Displacements 84
3.7 Geometrical Meanings of the Components of the Linear Strain Tensors 89
3.8 Principal Axis Theory for the Linear Strain Tensors 92
3.9 Additional Properties of Linear Strain Tensors 94
3.10 The Linear Cubical Dilatation 98
3.11 Compatibility Equations for Linear Strain Components 99
3.12 The Rate of Strain Tensor and the Vorticity Tensor 103
3.13 The Rate of Rotation Vector and the Vorticity Vector 105
3.14 Properties of the Rate of Strain Tensor 107
3.15 Rate of Cubical Dilatation 109
3.16 Analogies Among the Stress, Strain, Rotation, Rate of Strain, and Vorticity Tensors 109
Summary 113
Problems 115

CHAPTER 4

Eulerian Forms of the Basic Physical Laws Governing the Motion of a Continuous Medium 118

4.1 Introduction 118
4.2 Law of Conservation of Mass and the Eulerian Continuity Equation 122
4.3 The Momentum Integral Theorem and the Equation of Motion 123
4.4 Kinetic Equation of State 125
4.5 The First Law of Thermodynamics 128
4.6 The Second Law of Thermodynamics, Entropy Production, and the Dissipation Function 133
Summary 137

CHAPTER 5

Application to Solids 140

5.1 Introduction 140
Part I Linear Elasticity
5.2 Assumptions and Basic Equations 141
5.3 Generalized Hooke's Law for An Isotropic, Homogeneous Solid 146
5.4 Compatibility Equations Expressed in Terms of the Stress Components for an Isotropic, Homogeneous, Linear, Elastic Solid 152
5.5 Classification of Types of Problems in Linear Elasticity 153
5.6 The Principle of Superposition 155
5.7 The Strain Energy Function 156
5.8 The Uniqueness Theorem 160
5.9 Saint Venant's Principle 161
5.10 Constitutive Laws for Anisotropic Elastic Solids 162
Part II Thermoelasticity
5.11 Equations of State, the First Law of Thermodynamics, and the Coupled Heat Equation for an Isotropic, Homogeneous Thermoelastic Solid 166
5.12 Formulations of the Basic Equations 170
5.13 Irreversibility Due to Heat Conduction 172
Part III Viscoelasticity
5.14 Constitutive Equations 172
Summary 174
Problems 178

CHAPTER 6

Application to Fluids 181

6.1 Introduction 181
6.2 Method of Description 181
6.3 Separation of the Stress Tensor and Fluid Pressure 182
6.4 The Equation of Continuity 185
6.5 The Equations of Motion 187
6.6 Fluid Statics 190
6.7 Vorticity-Stream Surfaces for Inviscid Flow 193
6.8 p,ρ-Relationship and the Work-Kinetic Energy Equation 195
6.9 Irrotational Flow and the Velocity Potential 198
6.10 Kinetic Equation of State and the First Law of Thermodynamics 202
6.11 Second Law of Thermodynamics. Irreversibility Due to Material Dissipation and Heat Conduction 203
6.12 Similarity Parameters of Fluid Flow 205
Summary 207
Problems 209

Selected References 213

Index 215