## CONTENTS

1. THE FUNDAMENTAL EQUATIONS ..... 1
1.0. Introduction,
1.1. Fluid Motion, 1
1.2. The Jacobian, 5
1.3. Reynolds Transport Theorem, 13
1.4. Conservation of Mass, 13 ..... 13
1.5. Conservation of Momentum for a Perfect Fluid, 14 ..... 14
1.6. Conservation of Energy for a Perfect Fluid, 16 ..... 16
1.7. The Acoustic Wave Equation, ..... 22
1.8. Conservation of Energy in the Acoustic Approximation, 30
1.9. Boundary Conditions, 35
References, 37
2. MATHEMATICAL REVIEW ..... 39
2.0. Introduction, ..... 39
2.1. Separation of Variables, ..... 39
2.2. Sturm-Liouville Theory, ..... 44
2.3. Calculus of Variations, ..... 49
2.4. Eigenfunctions and the Variational Principle, ..... 57
2.5. Completeness of a Set of Eigenfunctions, ..... 59
2.6. The Continuous Spectrum, ..... 71
2.7. Green's Functions, ..... , 77
2.8. Method of Steepest Descent, 82
2.9. Bessel's Differential Equation of Arbitrary Order, 90
2.10. Airy's Differential Equation, 107
References, 110
3. PROPAGATION IN THE OCEAN AS A BOUNDARY VALUE PROBLEM: THE HOMOGENEOUS LAYERED MODEL ..... 111
3.0. Introduction, ..... 111
3.1. The Complex $\lambda$-Plane Representation of the Green's Function for a Homogeneous Layer Bounded by a Pressure-Release Surface and a Rigid Bottom, 111
3.2. Eigenfunction Expansion of the Green's Function for a Homogeneous Layer Bounded by a Pressure-Release Surface and a Rigid Bottom, 127
3.3. The Complex $\lambda$-Plane Representation of the Green's Function for an Ocean Consisting of Two Homogeneous Layers Bounded by a Pressure-Release Surface and a Rigid Bottom, 132
3.4. Eigenfunction Expansion of the Green's Function for an Ocean Consisting of Two Homogeneous Layers Bounded by a Pressure-Release Surface and a Rigid Bottom, 139
3.5. The Complex $\lambda$-Plane Representation of the Green's Function for an Ocean Consisting of a Homogeneous Layer with a Pressure-Release Surface Overlying a Homogeneous Halfspace-The Discrete Plus Continuous Spectrum, 144
Appendix 3.A. Diffraction Theory and the Sommerfeld RadiationCondition, 155
Appendix 3.B. Evaluation of the Complex Integral Solution of the Wave Equation Using the Contour $C_{r}^{+}, 160$
References, 162
4. PROPAGATION IN THE OCEAN AS A BOUNDARY VALUE PROBLEM: THE INHOMOGENEOUS LAYERED MODEL ..... 163
4.0. Introduction, ..... 163
4.1. A Normal Model for $N$-Inhomogeneous Layers with Discontinuous Properties, 163
Reference, ..... 177
CONTENTS ..... xi
5. APPROXIMATE SOLUTIONS OF THE WAVE EQUATION ..... 178
5.0. Introduction, ..... 178
5.1. Ray Equations as a Quasi-Plane Wave Approximation, ..... 178
5.2. WKB Approximation, ..... 197
5.3. Correction for Turning Points in the WKB Method, 204
5.4. WKB Solution in a Waveguide, ..... 211
5.5. WKB Green's Function for an Unbounded Media, 214
5.6. Ray Theory Approximation by the Method of SteepestDescent, 217
5.7. Physical Description of the Formation of a Smooth Caustic, ..... 220
5.8. Correction to Ray Theory for Smooth Caustics, ..... 227
Appendix 5.A. Derivation of the -90 -Degree Phase Shift as a Wave Front Passes Through a Focal Point, 235
References, 238
6. APPLICATION TO CONVERGENCE ZONE AND SURFACE DUCT PROPAGATION ..... 240
6.0. Introduction, ..... 240
6.1. Speed of Sound in the Ocean, ..... 240
6.2. Convergence Zone Propagation-A Single Channel NorthAtlantic Profile, 241
6.3. Convergence Zone Propagation-A Double Channel North Atlantic Profile, 26
6.4. Surface Duct Propagation-A North Atlantic Profile, ..... 273
References, ..... 293
7. AN OCEANIC WAVEGUIDE WITH A RANGE- AND DEPTH- DEPENDENT REFRACTIVE INDEX AND A TIME VARYING, RANDOMLY ROUGH SEA SURFACE ..... 294
7.0. Introduction, ..... 294
7.1. The Coupled Second-Order System of Differential Equations in Cylindrical Coordinates, ..... 294
7.2. The Coupled First-Order System of Differential Equationsin Cylindrical Coordinates, 302
7.3. A Model for a Time Varying, Randomly Rough Sea Surface, 305

# 7.4. The Coupled Second-Order System of Differential 

 Equations in Cartesian Coordinates, 3077.5. The Coupled First-Order System of Differential Equations
in Cartesian Coordinates, 312

Appendix 7.A. The Narrow-Band Approximation to the Wave
Equation, 315
References, 316
INDEX 317

