Contents

1. Introduction

1.1. Mathematical theories and engineering science, 1. 1.2. Scalars, vectors, and tensors, 3. 1.3. Preview, 6.

2. Cartesian Vectors and Tensors: Their Algebra

2.11. Definition of a vector, 8. 2.12. Examples of vectors, 10. 2.13. Scalar multiplication, 11. 2.21. Addition of vectors - Coplanar vectors, 11. 2.22. Unit vectors, 13. 2.23. A basis of non-coplanar vectors, 13. 2.31. Scalar product - Orthogonality, 15. 2.32. Vector product, 16. 2.33. Velocity due to rigid body rotation, 17. 2.34. Triple scalar product, 18. 2.35. Triple vector product, 19. 2.36. Reciprocal base systems, 20. 2.41. Second order tensors, 21. 2.42. Examples of second order tensors, 22. 2.43. Scalar multiplication and addition, 23. 2.44. Contraction and multiplication, 23. 2.45. The vector of an antisymmetric tensor, 24. 2.5. Canonical form of a symmetric tensor, 25. 2.61. Higher order tensors, 28. 2.62. The quotient rule, 29. 2.7. Isotropic tensors, 30. 2.81. Dyadics and other notations, 34. 2.82. Axial vectors, 36.

3. Cartesian Vectors and Tensors: Their Calculus

3.11. Tensor functions of a time-like variable, 38. 3.12. Curves in space, 39. 3.13. Line integrals, 42. 3.14. Surface integrals, 44. 3.15. Volume integrals, 48. 3.16. Change of variable with multiple integrals, 50. 3.21. Vector fields, 51. 3.22. The vector operator ∇ — Gradient of a scalar, 51. 3.23. The divergence of a vector field, 53. 3.24. The curl of a vector field, 55. 3.31. Green's theorem and some of its variants, 58. 3.32. Stokes' theorem, 61. 8

1

3.41. The classification and representation of vector fields, 63. 3.42. Irrotational vector fields, 65. 3.43. Solenoidal vector fields, 67. 3.44. Helmholtz' representation, 70. 3.45. Other representations, 72.

4. The Kinematics of Fluid Motion

4.11. Particle paths, 76. 4.12. Streamlines, 79. 4.13. Streaklines, 81. 4.21. Dilatation, 83. 4.22. Reynold's transport theorem, 84. 4.3. Conservation of mass and the equation of continuity, 87. 4.41. Deformation and rate of strain, 88. 4.42. Physical interpretation of the deformation tensor, 89. 4.43. Principal axes of deformation, 92. 4.5. Vorticity, vortex lines and tubes, 95.

5. Stress in Fluids

5.11. Cauchy's stress principle and the conservation of momentum, 99. 5.12. The stress tensor, 101. 5.13. The symmetry of the stress tensor, 102. 5.14. Hydrostatic pressure, 105. 5.15. Principal axes of stress and the notion of isotropy, 105. 5.21. The Stokesian fluid, 106. 5.22. Constitutive equations of the Stokesian fluid, 107. 5.23. The Newtonian fluid, 110. 5.24. Interpretation of the constants λ and μ , 111.

6. Equations of Motion and Energy in Cartesian Coordinates

6.11. Equations of motion of a Newtonian fluid, 113. 6.12. Boundary conditions, 115. 6.13. The Reynolds number, 115. 6.14. Dissipation of energy by viscous forces, 117. 6.2. Equations for a Stokesian fluid, 119. 6.3. The energy equation, 120. 6.41. Résumé of the development of the equations, 123. 6.42. Special cases of the equations, 124. 6.51. Bernoulli theorems, 131. 6.52. Some further properties of barotropic flow, 132.

7. Tensors

7.11. Coordinate systems and conventions, 134. 7.12. Proper transformations, 136. 7.13. General plan of presentation, 139. 7.21. Contravariant vectors, 140. 7.22. Covariant vectors, 141. 7.23. The metric tensor, 142. 7.24. Absolute and relative tensor fields, 144. 7.25. Isotropic tensors, 146. 7.31. Tensor algebra, 146. 7.32. The quotient rule, 148. 7.33 Length of a vector and angle between vectors, 149. 7.34. Principal directions of a symmetric second order tensor, 151. 7.35. Covariant and contravariant base vectors, 151. 7.41. Physical components of vectors in orthogonal coordinate systems, 153. 7.42. Physical components of vectors in nonorthogonal coordinate systems, 155. 7.43. Physical components of tensors, 156. 7.44. An 76

99

113

....

Contents

example, 157. 7.45. Anholonomic components of a tensor, 159. 7.51. Differentials of tensors, 160. 7.52. Parallel vector fields, 161. 7.53. Christoffel symbols, 162. 7.54. Christoffel symbols in orthogonal coordinates, 164. 7.55. Covariant differentiation, 166. 7.56. The Laplacian, divergence, and curl, 169. 7.57. Green's and Stokes' theorems, 171. 7.6. Euclidean and other spaces, 172.

8. Equations of Fluid Flow in Euclidean space

8.11. Intrinsic derivatives, 176. 8.12. The transport theorem and equation of continuity, 177. 8.13. The equations of motion, 178. 8.21. The Newtonian fluid, 180. 8.22. The Navier-Stokes equations, 181. 8.31. Convected coordinates, 183. 8.32. Convective differentiation, 185. 8.33. Strain and rate of strain in convected coordinates, 187. 8.34. Constitutive equations, 188. 8.4. The general theory of constitutive equations, 190.

9. The Geometry of Surfaces in Space

9.11. Surface coordinates, 193. 9.12. Transformations of surface coordinates — surface tensors, 194. 9.13. The metric tensor, 196. 9.14. Length and direction of surface vectors, 198. 9.21. Christoffel symbols, 199. 9.22. Geodesics, 201. 9.23. Geodesic coordinates, 204. 9.24. Parallel vectors in a surface, 206. 9.25. Covariant surface differentiation, 209. 9.26. The Gaussian or total curvature of a surface, 210. 9.31. The surface in space, 212. 9.32. The first fundamental form of the surface, 213. 9.33. The normal to the surface, 214. 9.34. Covariant differentiation of hybrid tensors, 215. 9.35. The second fundamental form of the surface, 216. 9.36. The third fundamental form, 217. 9.37. The relation between the three fundamental forms — Gauss-Codazzi equations, 218. 9.38. Curves in the surface, 219. 9.41. Differential operators in a surface, 222. 9.42. Green's and Stokes' theorems in a surface, 223.

10. The Equations of Surface Flow

10.11. Velocity in a surface, 227. 10.12. Operations with a time dependent metric, 228. 10.21. Strain in the surface, 230. 10.22. Stress in the surface, 231. 10.23. Constitutive relations for the surface, 232. 10.31. Intrinsic equations of surface motion, 233. 10.32. Intrinsic equations for a Newtonian surface fluid, 234. 10.41. The continuity of the surface and its surroundings, 235. 10.42. Connection between surface strain and the surroundings, 237. 10.43. Dynamical connection between the surface and its surroundings, 238. 10.51 Surface equations as boundary conditions at an interface, 241. 10.52. The plane interface, 242. 10.53. The cylindrical interface, 243. 10.54. The spherical interface, 243.

193

176

11. Equations for Reacting Fluids

11.11. The conservation of matter, 245. 11.12. Mass fluxes, 247. 11.13. Stoichiometric and kinetic relations, 248. 11.2. The conservation of momentum, 249. 11.31. The conservation of energy, 250. 11.32. The diffusion of heat and matter, 251. 11.33. Transport in binary mixtures, 252.

Appendix A. Résumé of Three-dimensional Coordinate Geometry and Matrix Theory

A.1. Cartesian coordinate systems, 254. A.2. The projection of one line on another — Orthogonality, 256. A.3. The line, plane, and surface, 257.
A.4. Row and column vectors — change of origin and scale, 258. A.5. Matrices and quadrics, 259. A.6. Matrices and rotations of axes, 262.
A.7. The laws of matrix algebra, 263. A.8. Determinants — the inverse of a matrix, 265. A.9. Partitioned matrices — Laplacian expansion — product of determinants, 268. A.10. Latent roots and vectors of a symmetric matrix, 270. A.11. Canonical form of symmetric matrices and quadrics, 271.
A.12. Stationary properties, 274.

Appendix B. Implicit Functions and Jacobians

Index

281

276