CONTENTS

CHAPTER PAGE

I. INTRODUCTION TO RESEARCH IN FLUID MOTION

A. Mathematical Analysis Versus Experimental Measurement

1. Present Status of the Science 1
2. Limitations of Mathematical Analysis 2
3. Limitations of Unguided Experiment 3
4. Method of Presentation 4

B. Dimensional Considerations

5. Dimensional Homogeneity 5
6. The Π-Theorem 7
7. Non-Dimensional Relationships and Their Significance 11

C. Investigational Procedure

8. Selection of Primary Variables 15
9. Organization of Parameters 16
10. Conduct of Investigation 17
11. Analysis of Results 19

D. Discussion of Typical Investigations

12. Problems of Mass Acceleration 21
13. Gravitational Phenomena 23
14. Flows with Viscous Resistance 25
15. Transportation of Discrete Matter 26

II. FUNDAMENTAL CONCEPTS AND EQUATIONS

A. Flow of a Continuous Medium

16. The Continuum 29
17. Velocity as a Function of Time and Space Coordinates 30
18. Equations of Continuity 32

B. Characteristics of the Flow Pattern

19. Stream Lines 37
20. Stream Functions of Lagrange and Stokes 39
21. Stream Surfaces in Three-Dimensional Flow 42

C. Kinematics of Flow

22. Translation, Deformation, Rotation 46
23. Vorticity and Circulation 49
24. Components of Acceleration 53

D. Dynamics of Flow

25. Forces on a Fluid Element 56
26. Equations of Motion 59
27. Equations of Momentum and Energy 63

CHAPTER PAGE

III. PRINCIPLES OF IRROTATIONAL FLOW

A. FUNDAMENTAL POTENTIAL THEORY
28. Characteristics of Potential Fields 68
29. Important Theorems and Equations 75
30. Classification of Irrotational-Flow Problems 79

B. TYPICAL FLOW SYSTEMS
31. Basic Potentials and Stream Functions 82
32. Combined Flows 92
33. Dynamic Relationships 94

C. MATHEMATICAL TECHNIQUES FOR SOLVING PROBLEMS
34. Separation of Variables 101
35. Method of Images 114
36. Method of Integral Equations 121

D. APPROXIMATION TECHNIQUES
37. Graphical Methods 130
38. Continuous-Conductor Analogs 133
39. Finite-Difference Principles 136

IV. CONFORMAL REPRESENTATION OF TWO-DIMENSIONAL FLOW

A. INTRODUCTION TO FUNCTIONS OF A COMPLEX VARIABLE
40. Functions of a Complex Variable 143
41. The Cauchy Integral Theorem 149
42. Application to Potential Flow 152

B. PRINCIPLES OF CONFORMAL MAPPING
43. Geometric Properties of Analytic Functions 158
44. Elementary Transformations 163

C. FLOWS WITH GIVEN BOUNDARIES
45. Flows about Single Obstacles 177
46. A Neumann Problem; Linearized Foil Theory 184
47. Flows about Two or More Obstacles 187
48. Polygonal Boundaries; Schwarz-Christoffel Mapping 189

D. FREE-STREAM-LINE FLOWS; THE HODOGRAPH METHOD
49. Free-Stream-Line Theory 192
50. Applications to Efflux and Jet Deflection 195

V. LAMINAR MOTION

A. FUNDAMENTALS
51. Relationship between Stress and Rate of Deformation 200
52. Physical Interpretations of Viscosity 204
53. Rate of Energy Dissipation 205

B. THE NAVIER-STOKES EQUATIONS
54. Formulation of the Equations 207
55. Equations Governing the Diffusion of Vorticity 210

C. EXACT SOLUTIONS OF THE NAVIER-STOKES EQUATIONS
56. Steady Linear Cases 215
57. Unsteady Linear Cases 221
58. Non-Linear Cases 224

CHAPTER PAGE

D. Approximate Solutions of the Navier-Stokes Equations
59. Very Slow Motion—Stokes' Solution for the Falling Sphere 236
60. Laminar Wakes 240
61. Non-Linear Cases: Jets 242

E. Stability of Laminar Flow
62. General Remarks 248
63. The Method of the Exponential Time Factor 250
64. Sufficient Conditions for Stability 254
65. The Method of Energy and the Method of Vorticity 256
66. Effect of Nature of Disturbance on Stability 258

VI. TURBULENCE

A. General Considerations
67. Introductory Remarks 261
68. Mean Values 261
69. Probability-Density Functions 264

B. Fundamental Equations for Turbulent Flows
70. Reynolds Equations 267
71. Equation of Linear Momentum 271
72. Equation of Energy 272

C. Isotropic Turbulence
73. Definitions and Descriptive Relationships 276
74. Dynamics of Isotropic Turbulence 283
75. Frequency Distribution of the Kinetic Energy 286

D. Turbulent Diffusion
76. Descriptive Relationships 291
77. The Diffusion Equation 293

E. Turbulent Shear Flow
78. Phenomenological Theories 297
79. Typical Measurements in Shear Flows 300

VII. BOUNDARY LAYERS

A. Basic Concepts
80. Nature of Flow along a Solid Boundary 306
81. Assumptions of Boundary-Layer Theory 309
82. Definitions of Boundary-Layer Thicknesses 309

B. Boundary-Layer Equations and Their Integrals
83. Two-Dimensional Flow along a Curved Wall 312
84. Axisymmetric Flow about a Surface of Revolution 319
85. Three-Dimensional Flow along an Arbitrary Surface 322

C. Laminar Boundary Layers
86. Solutions for Steady Two-Dimensional Flows 325
87. Solutions for Steady Axisymmetric Flow 333
88. Approximate Methods of Solution for Steady Flows 336
89. Unsteady Boundary Layers 339

D. Turbulent Boundary Layers
90. Mean Velocity Distribution Near the Wall 343

CHAPTER PAGE

91. Mean Velocity Distribution at a Distance from the Wall 346
92. Overlapping of Inner and Outer Laws 348
93. Boundary Layer on a Flat Plate 352
94. Approximate Calculation of Turbulent Boundary Layers 356

VIII. FREE-TURBULENCE SHEAR FLOW

A. Introductory Remarks
95. Flow Induced by a Velocity Discontinuity 359
96. Methods of Analysis 362

B. The Mixing of Parallel Streams
97. General Relationships for Mixing Streams 364
98. The Velocity Distribution of Mixing Streams 368

C. The Spreading of Wakes
99. Integral Relationships for Wakes 371
100. General Characteristics of Wakes 375
101. Velocity Distribution in Wakes 378

D. Diffusion of Jets
102. Integral Relationships for Jets 382
103. Velocity Distribution in the Jet 385

E. Comparison between Theory and Observation
104. Completeness of Analyses 389
105. Mean-Flow Characteristics of Jets and Wakes 390
106. Turbulence Characteristics of Jets 394

F. Extension of Analyses to Finite Fields of Flow
107. Fundamental Difference between Finite and Infinite Fields 400
108. Examples of Analyses for Finite Fields of Flow 402

APPENDIX

A. Equations in Curvilinear Orthogonal Coordinates 407
B. List of Symbols 420

ANSWERS TO PROBLEMS 424

AUTHOR INDEX 437

SUBJECT INDEX 439