

# Contents

<b>Preface</b> .....	ix
<b>1 Introduction</b> .....	1
1.1 Historical background .....	1
1.2 Chaotic dynamics in Duffing's oscillator .....	3
1.3 Attractors and bifurcations .....	9
<b>PART I BASIC CONCEPTS OF NONLINEAR DYNAMICS</b>	
<b>2 An overview of nonlinear phenomena</b> .....	15
2.1 Undamped, unforced linear oscillator .....	15
2.2 Undamped, unforced nonlinear oscillator .....	17
2.3 Damped, unforced linear oscillator .....	19
2.4 Damped, unforced nonlinear oscillator .....	20
2.5 Forced linear oscillator .....	21
2.6 Forced nonlinear oscillator: periodic attractors .....	23
2.7 Forced nonlinear oscillator: strange attractor .....	25
<b>3 Point attractors in autonomous systems</b> .....	27
3.1 The linear oscillator .....	27
3.2 Nonlinear pendulum oscillations .....	36
3.3 Evolving ecological systems .....	44
3.4 Competing point attractors .....	49
<b>4 Limit cycles in autonomous systems</b> .....	51
4.1 The single attractor .....	51
4.2 Limit cycle in a neural system .....	52
4.3 Bifurcations of a chemical oscillator .....	56
4.4 Multiple limit cycles in aeroelastic galloping .....	60
4.5 Topology of two-dimensional phase space .....	63
<b>5 Periodic attractors in driven oscillators</b> .....	65
5.1 The Poincaré map .....	65

5.2	Linear resonance .....	67
5.3	Nonlinear resonance .....	69
5.4	The smoothed variational equation .....	74
5.5	Variational equation for subharmonics .....	76
5.6	Domains of attraction by mapping techniques .....	77
5.7	Resonance of a self-exciting system .....	80
<b>6</b>	<b>Chaotic attractors in forced oscillators .....</b>	<b>84</b>
6.1	Relaxation oscillations and heartbeat .....	84
6.2	The Birkhoff–Shaw chaotic attractor .....	87
6.3	Systems with nonlinear restoring force .....	98
<b>7</b>	<b>Stability and bifurcations of equilibria and cycles .....</b>	<b>108</b>
7.1	Liapunov stability and structural stability .....	108
7.2	Centre manifold theorem .....	111
7.3	Local bifurcations of equilibrium paths .....	113
7.4	Local bifurcations of cycles .....	125
7.5	Prediction of incipient instability .....	128

## **PART II ITERATED MAPS AS DYNAMICAL SYSTEMS**

<b>8</b>	<b>Stability and bifurcation of maps .....</b>	<b>135</b>
8.1	Introduction .....	135
8.2	Stability of one-dimensional maps .....	138
8.3	Bifurcations of one-dimensional maps .....	139
8.4	Stability of two-dimensional maps .....	150
8.5	Bifurcations of two-dimensional maps .....	158
<b>9</b>	<b>Chaotic behaviour of one- and two-dimensional maps .....</b>	<b>162</b>
9.1	General outline .....	162
9.2	Theory for one-dimensional maps .....	165
9.3	Bifurcations to chaos .....	170
9.4	Bifurcation diagram of one-dimensional maps .....	173
9.5	Hénon map .....	177

## **PART III FLOWS, OUTSTRUCTURES, AND CHAOS**

<b>10</b>	<b>The geometry of recurrence .....</b>	<b>187</b>
10.1	Finite-dimensional dynamical systems .....	187
10.2	Types of recurrent behaviour .....	192
10.3	Hyperbolic stability types for equilibria .....	200
10.4	Hyperbolic stability types for limit cycles .....	206
10.5	Implications of hyperbolic structure .....	211

<b>11</b>	<b>The Lorenz system</b> .....	212
	11.1 A model of thermal convection .....	212
	11.2 First convective instability .....	215
	11.3 The chaotic attractor of Lorenz .....	219
	11.4 Geometry of a transition to chaos .....	227
<b>12</b>	<b>Rössler's band</b> .....	235
	12.1 The simply folded band in an autonomous system .....	235
	12.2 Return map and bifurcations .....	239
	12.3 Smale's horseshoe map .....	245
	12.4 Transverse homoclinic trajectories .....	251
<b>13</b>	<b>Geometry of bifurcations</b> .....	254
	13.1 Local bifurcations .....	254
	13.2 Global bifurcations in the phase plane .....	263
	13.3 Bifurcations of chaotic attractors .....	272
 <b>PART IV APPLICATIONS IN THE PHYSICAL SCIENCES</b>		
<b>14</b>	<b>Subharmonic resonances of an offshore structure</b> .....	291
	14.1 Basic equation and non-dimensional form .....	292
	14.2 Analytical solution for each domain .....	294
	14.3 Digital computer program .....	295
	14.4 Resonance response curves .....	296
	14.5 Effect of damping .....	301
	14.6 Computed phase projections .....	302
	14.7 Multiple solutions and domains of attraction .....	305
<b>15</b>	<b>Chaotic motions of an impacting system</b> .....	310
	15.1 Resonance response curve .....	310
	15.2 Application to moored vessels .....	314
	15.3 Period-doubling and chaotic solutions .....	316
<b>16</b>	<b>The particle accelerator and Hamiltonian dynamics</b> .....	321
	16.1 The physical model .....	322
	16.2 The mathematical model .....	323
	16.3 Resonance and Arnold diffusion .....	324
	16.4 The standard map .....	327
<b>17</b>	<b>Experimental observations of order and chaos</b> .....	332
	17.1 Introduction .....	332
	17.2 Four nonlinear systems .....	333
	17.3 Analysis of dynamical behaviour .....	335
	17.4 Transition sequences .....	340
	17.5 Discussion .....	347

<b>References and Bibliography</b> .....	350
<b>Index</b> .....	370