Contents

Preface v

1 Introduction, Motivation, and Background 1

1.1 Examples of the Use of Invariant Manifold Ideas and Methods in Applications 1

1.2 Issues and Methods 4

1.2.1 Existence of Invariant Manifolds 5

1.2.2 The Persistence and Differentiability of Invariant Manifolds Under Perturbaton 5

1.2.3 Behavior Near an Invariant Manifold—Stable, Unstable, and Center Manifolds 6

1.2.4 More Refined Behavior Near Invariant Manifolds—Foliations of Stable, Unstable, and Center Manifolds 7

1.2.5 The Liapunov–Perron Method 7

1.2.6 Hadamard's Method—The Graph Transform 8

1.2.7 The Lie Transform, or Deformation Method 8

1.2.8 Irwin's Method 9

1.3 Motivational Examples 9

2 Background from the Theory of Differentiable Manifolds 21

2.1 The Definition of a "Manifold" and Examples 21

2.2 Derivatives, Tangent Spaces, Normal Spaces, and Other "Structures" on Manifolds 28

2.2.1 The Normal Space at a Point 33

2.2.2 Tangent Bundles and Normal Bundles 34

2.2.3 The Neighborhood of a Manifold 36

2.3 Manifolds with Boundary 39

2.4 An Example 41

3 Persistence of Overflowing Invariant Manifolds—Fenichel's Theorem 51

3.1 Overflowing Invariant Manifolds and Generalized Lyapunov-Type Numbers 56

3.1.1 "Stability" of M and Generalized Lyapunov-Type Numbers 57

3.1.2 Some Properties of Generalized Lyapunov-Type Numbers 62
3.2 Coordinates and Dynamics Near $\bar{M}$ 70
3.2.1 Local Coordinates Near $\bar{M}$. 70
3.2.2 "Jiggling" the Normal Bundle to Gain a Derivative, and the Construction of a Neighborhood of M_1 . . . 72
3.2.3 Local Expressions for the Time-T Map Generated by the Flow 74
3.2.4 The Perturbed Vector Field and Flow 79
3.2.5 Smallness and Closeness Parameters 81
3.2.6 The Space of Sections of the Transversal Bundle . . 81
3.2.7 The Graph Transform 81
3.3 Statement and Proof of the Main Theorem 85
3.3.1 Differentiability of the Persisting Overflowing Invariant Manifold 92

4 The Unstable Manifold of an Overflowing Invariant Manifold 111
4.1 Generalized Lyapunov-Type Numbers 111
4.2 Local Coordinates Near M 114
4.3 The GR-I Estimates 116
4.4 The Space of Sections of $N'_\epsilon|_{\cup_{i=1}^s U_i^3}$ over $N'^u_\epsilon|_{\cup_{i=1}^s U_i^3}$ 119
4.5 The Unstable Manifold Theorem 120
4.5.1 Differentiability of $W^u(\bar{M})$ 128
4.5.2 $W^u(\bar{M})$ Satisfies the Hypotheses of the Persistence Theorem for Overflowing Invariant Manifolds 129

5 Foliations of Unstable Manifolds 131
5.1 Generalized Lypaunov-Type Numbers 131
5.2 Local Coordinates Near M 134
5.3 The GR-I Estimates 134
5.4 The Space of Families of u-Dimensional Lipschitz Surfaces . 135
5.5 Sequences of Contraction Maps 136
5.6 The Theorem on Foliations of Unstable Manifolds 138
5.6.1 The Proof of the Theorem 139
5.7 Persistence of the Fibers Under Perturbations 157

6 Miscellaneous Properties and Results 159
6.1 Inflowing Invariant Manifolds 159
6.2 Compact, Boundaryless Invariant Manifolds 159
6.3 Boundary Modifications 160
6.4 Parameter-Dependent Vector Fields 162
6.5 Continuation of Overflowing Invariant Manifolds— The "Size" of the Perturbation 163
6.6 Discrete Time Dynamics, or "Maps" 163

7 Examples **165**
7.1 Invariant Manifolds Near a Hyperbolic Fixed Point 165
7.2 Invariant Manifolds Near a Nonhyperbolic Fixed Point . . . 166
7.3 Weak Hyperbolicity 168
7.4 Asymptotic Expansions for Invariant Manifolds 170
7.5 The Invariant Manifold Structure Associated with the Study of Orbits Homoclinic to Resonances 171
7.5.1 The Analytic and Geometric Structure of the Unperturbed Equations 172
7.5.2 The Analytic and Geometric Structure of the Perturbed Equations 174

References **185**

Index **191**