

Contents

CHAPTER I. INTRODUCTION TO BROWNIAN MOTION

1. WIENER MEASURE (§§ 1–29)	1
Wiener's theorem (§ 6); A converse (9); Diffusions (10); Canonical and non-canonical processes (11–14); Martingale descriptions of Brownian motion (15–17); Lévy's theorem (17); Trotter's theorem (18–20); Local and global properties (21–26); Blumenthal's 01 law (23); Kolmogorov's test (24); Iterated-logarithm law (25); Hölder condition (26); Completions, almost surely (P) (27–29).	
2. NARROW CONVERGENCE (30–52)	18
Donsker invariance principle (30–39); Polish spaces (36); Narrow convergence of measures (37); Prohorov's theorem (40–47); Narrow convergence in $\text{Pr}(W)$ (48–52).	
3. BROWNIAN MOTION IN \mathbb{R}^n (53–66)	29
Potential theory (58–62); Equilibrium potential (59); Bessel processes (63–66); Skew product (64); Ray–Knight theorem on local time (65).	

CHAPTER II. SOME CLASSICAL THEORY

1. BASIC MEASURE THEORY (1–14)	39
Monotone-class theorems (1–5, 13–14); Daniell–Kolmogorov theorem (6); Its limitations (7); Fubini's theorem (8); Infinite products (9); Poisson measures (10); Stochastic process (11); Modification (12).	
2. CLASSICAL MARTINGALE THEORY (15–55)	50
Uniform integrability (15–20); Conditional expectations and probabilities (21–24); Discrete-parameter martingales (23–34); Continuous-parameter supermartingales (35–55); Basic definitions (36); Skorokhod (càdlàg) maps (37); Doob's regularity theorem (38–41); The ‘usual conditions’ (40); Indistinguishability, evanescence (42); Inequalities and the convergence theorem (43); Stopping	

times (44–50); Début and section theorems (51–52); Stopping times and supermartingales (53–55).	
3. APPLICATIONS (56–67)	81
Proof of Wiener's theorem (56); Strong Markov theorem for Brownian motion (57); Hitting-times, reflection principle, etc. (58–61); Lévy's downcrossing theorem (62); Local time (63); Hitting-time process as subordinator (64); Pitman's presentation of 3-dimensional Bessel process (65); Excursion theory (66–67).	
4. REGULAR CONDITIONAL PROBABILITIES (68–72)	100
Main theorem (69); Fundamental statements of Markov property (72).	

CHAPTER III. MARKOV PROCESSES

1. TRANSITION FUNCTIONS AND RESOLVENTS (1–6)	106
Definitions (1–2); Hille–Yosida theorems (3–6).	
2. FELLER TRANSITION FUNCTIONS (7–10)	114
Feller–Dynkin (FD) semigroups (8); Dynkin's maximum principle (9).	
3. FELLER–DYNKIN PROCESSES (11–31)	117
Path regularization (13); Canonical FD process (14); Strong Markov theorem for FD processes (15–17); Completions (16); Blumenthal's 01 law (18); Some fundamental martingales, Dynkin's formula (22); Quasi-left-continuity (23); Characteristic operator (24); FD diffusions (26–29); Dirichlet problem (30).	
4. ADDITIVE FUNCTIONALS (32–42)	141
Some basic facts about PCHAFs (32–34); Killing (35–36); Time-substitution (37); Volkonskii's formula, Arcsine law, Feller–McKean chain (38); Feynman–Kac formula (39); A Ciesielski–Taylor theorem (40); Elastic Brownian motion (41).	
5. RAY PROCESSES (43–66)	162
Motivation (43–54); Martin boundary theory for discrete-parameter chains (48); Probabilistic Doob–Hunt theory (discrete-parameter chains) (49); R. S. Martin's boundary (50); Doob–Hunt theory for Brownian motion (51); Ray's theorem: preparatory remarks (55–56); Ray–Knight compactification (57); Ray resolvents (58); Ray's theorem: analytic part (59–62); Branch-points (60); Ray's theorem: probabilistic part (63–66); Strong Markov theorem for Ray processes (64); The rôle of branch-points (65).	

6. APPLICATIONS (67–91)	198
Martin boundary theory in retrospect (68–73); Proof of the Doob–Hunt convergence theorem (70); Choquet representation of excessive functions (71); Doob’s h -transforms (72); Time-reversal and related topics (74–79); Nagasawa’s formula (75); Strong Markov property under time-reversal (76); Equilibrium charge (77); Splitting-times (79); A first look at chain theory (80–91); Chains as Ray processes (81); Taboo probabilities; first-entrance decompositions (83); The Q -matrix: DK conditions (84); Local character condition for Q (85); Totally instantaneous Q -matrices (86); Last exits (87); Excursions (88); Kingman’s solution of the Markov characterization problem (89); Q -matrix problem: symmetrizable case (90).	
REFERENCES	229
INDEX	235