



# Contents

<i>Preface</i> .....	ix
<i>Acknowledgments</i> .....	xi
<b>Introduction</b> .....	1

## CHAPTER ONE

### The Variational Characterization of Eigenvalues

1. A Preliminary Survey of the Classical and Minimax Principles for Eigenvalues .....	4
2. Operators of Class $\mathcal{S}$ .....	5
3. Rayleigh's Principle and the Classical Characterization .....	6

## CHAPTER TWO

### The Rayleigh-Ritz Method

1. Poincaré's Inequalities: The Theoretical Foundation of the Rayleigh-Ritz Method .....	10
2. The Minimum-Maximum Principle .....	11
3. Upper Bounds for Eigenvalues .....	13
4. A Necessary and Sufficient Criterion in the Minimum-Maximum Theory..	16
5. The Principle of Monotonicity .....	21

**CHAPTER THREE****The Classical Maximum-Minimum Theory and Its Extension to Unbounded Operators**

1. Weyl's First Fundamental Lemma.....	23
2. The Maximum-Minimum Principle.....	24
3. The Existence of Minima for Semibounded Operators.....	27
4. Comparison of the Minimum-Maximum and Maximum-Minimum Principles.....	30
5. Finite-Dimensional Spaces.....	32
6. The General Maxi-Mini-Max Principle.....	34

**CHAPTER FOUR****Intermediate Problems of The First Type**

1. Weinstein's General Scheme of Intermediate Problems.....	36
2. The Basic Principles of Intermediate Problems of the First Type.....	37
3. Nonpersistent Eigenvalues. Weinstein's Determinant.....	39
4. The Distinguished Choice.....	44
5. Lower Bounds Using a Distinguished Choice.....	49
6. The Existence of Distinguished Choices.....	50
7. The General Choice.....	52
8. Aronszajn's Rule for the Theoretical Determination of Eigenvalues.....	55
9. Comparison of the Various Rules.....	60
10. Weinberger-Bazley-Fox Method of Truncation.....	62
11. Convergence.....	63
12. Computation of Lower Bounds for the Buckling of a Clamped Plate.....	67
13. On the Symmetries of the Eigenfunctions.....	74
14. Weyl's Second Lemma.....	76

**CHAPTER FIVE****Intermediate Problems of The Second Type**

1. Formulation of Problems of the Second Type.....	79
2. Finite Rank Perturbations and Intermediate Problems of the Second Type..	82
3. A Solution of the Second Type.....	85
4. Bazley's Special Choice.....	89
5. The General Choice and Truncation.....	93
6. The Existence of a Base Problem for a Compact Operator.....	94
7. The Spectrum of the Helium Atom.....	96
8. Application of the Special Choice to the Helium Atom.....	102
9. Application of Intermediate Problems to Temple's Formula.....	103
10. Application of Truncation to Quantum Theory.....	104

**CHAPTER SIX****Various Other Methods and Their Connections  
with Intermediate Problems**

1. Quadratic Forms and Intermediate Problems.....	107
2. Application of Quadratic Forms to Free and Cantilever Plates.....	109
3. The Rhombical Membrane.....	113
4. Bazley-Fox Method for Sums of Solvable Operators.....	116
5. The Trefftz-Fichera Method for Integral Operators.....	118
6. Fichera's Construction of Intermediate Green's Operators.....	119

**CHAPTER SEVEN****The New Maximum-Minimum Theory**

1. The Basis of Weinstein's New Maximum-Minimum Theory.....	123
2. The Case of One Constraint.....	125
3. The Case of Orthonormal Constraints.....	130
4. The General Formulation.....	132
5. A Special Property of Operators Having Finite Traces.....	135
6. The Existence of a Nonclassical Choice.....	135
7. A Stability Problem for Viscous Fluids.....	137
8. Some Inequalities for Higher Eigenvalues.....	141

**CHAPTER EIGHT****Inequalities for Eigenvalues of Parts and Projections  
of Operators**

1. Some Preliminary Lemmas.....	146
2. Aronszajn's Inequality.....	149
3. An Estimate of the Rate of Convergence for Intermediate Problems .....	153
4. Stenger's Inequality .....	154
5. A Procedure for Obtaining Upper Bounds.....	156
6. Inequalities for Sums of Eigenvalues.....	157
7. Diaz and Metcalf's Generalized Inequalities.....	161
8. An Inequality for Sums of Operators.....	163

**CHAPTER NINE****Intermediate Problems and Perturbation Theory**

1. Summary of Previous Results.....	166
2. Two Optimum Problems for Positive Perturbations of Finite Rank .....	167
3. Kuroda's Generalization of the Weinstein-Aronszajn Determinant .....	170

4. Connections between Two Types of Intermediate Problems and the Irreducible Case .....	175
5. A Transformation for Positive Operators.....	177
6. Other Theoretical Results.....	180
 <b>Appendix A</b> .....	 181
 <b>Appendix B</b> .....	 193
 <b>Bibliography</b>	
Books and Monographs .....	221
Articles .....	222
 <i>Notation Index</i> .....	 233
<i>Subject Index</i> .....	234

