CONTENTS

	Page
Foreword	v
Chapter I. The Steady-State Stokes Equations.	1
Introduction	1
§1. Some function spaces	1
1.1. Notation	1
1.2 A density theorem	5
1.3 A trace theorem	9
1.4 Characterization of the spaces H and V	13
§2. Existence and uniqueness for the Stokes equations	20
2.1 Variational formulation of the problem	21
2.2 The projection theorem	23
2.3 The unbounded case	29
2.4 The non-homogeneous Stokes problem	31
2.5 Regularity results	32
2.6 Eigenfunctions of the Stokes problem	38
§3. Discretization of the Stokes equations (I)	39
3.1 Approximation of a normed space	40
3.2 A general convergence theorem	44
3.3 Approximation by finite differences	48
§4. Discretization of the Stokes equations (II)	65
4.1 Preliminary results	66
4.2 Finite elements of degree 2 ($n = 2$)	74
4.3 Finite elements of degree 3 ($n = 3$)	99
4.4 An internal approximation of V	104
4.5 Nonconforming finite elements	112
§5. Numerical algorithms	138
5.1 Uzawa algorithm	138
5.2 Arrow–Hurwicz algorithm	142
5.3 Discrete form of these algorithms	145

Page

§6. **Slightly compressible fluids** 147

6.1 Convergence of u_ϵ to u 148
6.2 Asymptotic expansion of u_ϵ 151
6.3 Numerical algorithms 154

Chapter II. The Steady-State Navier–Stokes Equations 157

Introduction 157

§1. **Existence and uniqueness theorems** 157

1.1 Sobolev inequalities and compactness theorems 158
1.2 The homogeneous Navier–Stokes equations 160
1.3 The homogeneous Navier–Stokes equations (continued) 168
1.4 The non-homogeneous Navier–Stokes equations 173

§2. **Discrete inequalities and compactness theorems** 180

2.1 Discrete Sobolev inequalities 180
2.2 A discrete compactness theorem for step functions 187
2.3 Discrete Sobolev Inequalities for non conforming finite elements 192
2.4 A discrete compactness theorem for non conforming finite elements 196

§3. **Approximation of the stationary Navier–Stokes equations** 199

3.1 A general convergence theorem 199
3.2 Applications 205
3.3 Numerical algorithms 218

§4. **Bifurcation theory and non-uniqueness results** 223

4.1 The Taylor problem. Preliminary Results 224
4.2 A spectral property of B 234
4.3 Elements of the topological degree theory 242
4.4 The non uniqueness theorem 244

Chapter III. The Evolution Navier–Stokes Equations 247

Introduction 247

Page

§1. **The linear case** 247

1.1 Notations 248
1.2 The existence and uniqueness theorem 252
1.3 Proof of the existence in Theorem 1.1 255
1.4 Proof of the continuity and uniqueness 260
1.5 Miscellaneous remarks 264

§2. **Compactness theorems** 269

2.1 A preliminary result 270
2.2 A compactness theorem in Banach spaces 271
2.3 A compactness theorem involving fractional derivatives 273

§3. **Existence and uniqueness theorems** ($n \leqslant 4$). 278

3.1 An existence theorem in $\mathcal{R}^n$ ($n \leqslant 4$) 279
3.2 Proof of Theorem 3.1 283
3.3 Regularity and uniqueness ($n = 2$) 291
3.4 About regularity and uniqueness ($n = 3$) 295
3.5 More regular solutions 299
3.6 Relations between the problems of existence and uniqueness ($n = 3$) 308
3.7 Utilization of a special basis 313
3.8 The special case $f = 0$ 318

§4. **Alternate proof of existence by semi-discretization** 320

4.1 Statement of the problem 320
4.2 The approximate solutions 323
4.3 *A priori* estimates 325
4.4 Passage to the limit 328

§5. **Discretization of the Navier–Stokes equations: General stability and convergence theorems** 331

5.1 Description of the approximation schemes 332
5.2 Stability of the Schemes 5.1 and 5.2 336
5.3 Stability of Scheme 5.3 340

		Page
5.4	Stability of Scheme 5.4	344
5.5	A complementary estimate for Scheme 5.2	348
5.6	Other *a priori* estimates	349
5.7	Convergence of the numerical schemes	352
§6.	**Discretization of the Navier–Stokes equations: Application of the general results**	364
6.1	Finite Differences (APX1)	365
6.2	Finite Elements (APX2), (APX3), (APX4)	374
6.3	Non Conforming finite elements (APX5)	381
6.4	Numerical algorithms. Approximation of the pressure	386
§7.	**Approximation of the Navier–Stokes equations by the Fractional Step Method**	395
7.1	A Scheme with two intermediate steps	397
7.2	A Scheme with $n + 1$ intermediate steps	409
7.3	Convergence of the Scheme	417
§8.	**Approximation of the Navier–Stokes equations by the artificial compressibility method**	426
8.1	Study of the perturbed problems	427
8.2	Convergence of the perturbed problems to the Navier–Stokes equations	440
8.3	Approximation of the perturbed problems	443
Comments		458
References		464
Appendix *(by F. Thomasset)*		
Implementation of non-conforming linear finite elements		480