



# Contents

	Page
Preface	v
Chapter 1. INTRODUCTION AND SIMPLE PROPERTIES	1
1.1. Introduction	1
1.2. Chaotic Ordinary Differential Equations	4
1.3. Our Approach to the Lorenz Equations	6
1.4. Simple Properties of the Lorenz Equations	8
Chapter 2. HOMOCLINIC EXPLOSIONS: THE FIRST HOMOCLINIC EXPLOSION	13
2.1. Existence of a Homoclinic Orbit	13
2.2. The Bifurcation Associated with a Homoclinic Orbit	16
2.3. Summary and Some General Definitions	22
Chapter 3. PRETURBULENCE, STRANGE ATTRACTORS AND GEOMETRIC MODELS	26
3.1. Periodic Orbits for the Hopf Bifurcation	26
3.2. Preturbulence and Return Maps	28
3.3. Strange Attractor and Homoclinic Explosions	32
3.4. Geometric Models of the Lorenz Equations	43
3.5. Summary	49
Chapter 4. PERIOD DOUBLING AND STABLE ORBITS	51
4.1. Three Bifurcations Involving Periodic Orbits	51
4.2. $99.524 < r < 100.795$ . The $x^2y$ Period Doubling Window	56
4.3. $145 < r < 166$ . The $x^2y^2$ Period Doubling Window	59
4.4. Intermittent Chaos	62
4.5. $214.364 < r < \infty$ . The Final $xy$ Period Doubling Window	66
4.6. Noisy Periodicity	69
4.7. Summary	74
Chapter 5. FROM STRANGE ATTRACTOR TO PERIOD DOUBLING	76
5.1. Hooked Return Maps	76
5.2. Numerical Experiments	79
5.3. Development of Return Maps as $r$ Increases: Homoclinic Explosions and Period Doubling	86
5.4. Numerical Experiments on Periodic Orbits	92
5.5. Period Doubling and One-Dimensional Maps	100
5.6. Global Approach and Some Conjectures	103
5.7. Summary	113
Chapter 6. SYMBOLIC DESCRIPTION OF ORBITS: THE STABLE MANIFOLDS OF $C_1$ AND $C_2$	115
6.1. The Maxima-in-z Method	116
6.2. Symbolic Descriptions from the Stable Manifolds of $C_1$ and $C_2$	119
6.3. Summary	130

	Page
<b>Chapter 7. LARGE r</b>	<b>132</b>
7.1. The Averaged Equations	132
7.2. Analysis and Interpretation of the Averaged Equations	136
7.3. Anomalous Periodic Orbits for Small b and Large r	140
7.4. Summary	149
<b>Chapter 8. SMALL b</b>	<b>151</b>
8.1. Twisting Around the z-Axis	151
8.2. Homoclinic Explosions with Extra Twists	153
8.3. Periodic Orbits Without Extra Twisting Around the z-Axis	158
8.4. Heteroclinic Orbits Between $C_1$ and $C_2$	164
8.5. Heteroclinic Bifurcations	169
8.6. General Behaviour When $b = 0.25$	171
8.7. Summary	176
<b>Chapter 9. OTHER APPROACHES, OTHER SYSTEMS, SUMMARY AND AFTERWORD</b>	<b>179</b>
9.1. Summary of Predicted Bifurcations for Varying Parameters $\sigma$ , $b$ and $r$	179
9.2. Other Approaches	184
9.3. Extensions of the Lorenz System	186
9.4. Afterword - A Personal View	189
<b>Appendix A. DEFINITIONS</b>	<b>192</b>
<b>Appendix B. DERIVATION OF THE LORENZ EQUATIONS FROM THE MOTION OF A LABORATORY WATER WHEEL</b>	<b>194</b>
<b>Appendix C. BOUNDEDNESS OF THE LORENZ EQUATIONS</b>	<b>196</b>
<b>Appendix D. HOMOCLINIC EXPLOSIONS</b>	<b>199</b>
<b>Appendix E. NUMERICAL METHODS FOR STUDYING RETURN MAPS AND FOR LOCATING PERIODIC ORBITS</b>	<b>211</b>
<b>Appendix F. COMPUTATIONAL DIFFICULTIES INVOLVED IN CALCULATING TRAJECTORIES WHICH PASS CLOSE TO THE ORIGIN</b>	<b>221</b>
<b>Appendix G. GEOMETRIC MODELS OF THE LORENZ EQUATIONS</b>	<b>223</b>
<b>Appendix H. ONE-DIMENSIONAL MAPS FROM SUCCESSIVE LOCAL MAXIMA IN z</b>	<b>234</b>
<b>Appendix I. NUMERICALLY COMPUTED VALUES OF <math>k(r)</math> FOR <math>\sigma = 10</math> and <math>b = 8/3</math></b>	<b>239</b>
<b>Appendix J. SEQUENCES OF HOMOCLINIC EXPLOSIONS</b>	<b>244</b>
<b>Appendix K. LARGE r; THE FORMULAE</b>	<b>257</b>
<b>Bibliography</b>	<b>262</b>

