CONTENTS

Preface			VII		
1.	Тне	Fourier-Laplace integral	1		
ARESAUT		The Laplace transform	1		
	- L L L L L L L L.	1.1.1. The direct problem	1		
		1.1.2. The inverse problem	4		
		1.1.3. Elementary rules	6		
	1.2.	The Fourier transform in L^1	10		
	1.3.	The Fourier transform in L^2	14		
	1.4.	The Laplace transform (continued)	20		
	1.5.	The Mellin transform	24		
2.	SPEC	IAL FUNCTIONS	29		
	2.I.	The gamma function	29		
		2.I.1. A summation formula	29		
		2.I.2. The Eulerian definition of the function $\Gamma(z)$	32		
		2.I.3. The Laplace transform of t^{ν}	33		
		2.I.4. The relation between the function $\Gamma(z)$ and the group of linear mappings			
		of the real line into itself	34		
		2.I.4.1. The Γ function and addition formulas	36		
		2.I.5. The error function	38		
	2.II.	The Bessel functions	39		
3/4		2.II.1. Definitions	39		
		2.II.1.1. The Bessel functions $J_{\frac{1}{2}}$, $J_{-\frac{1}{2}}$	42		
102		2.II.1.2. Some elementary properties of $J_{\nu}(t)$	42		
		2.II.2. The Kepler equation and Bessel functions	44		
		2.II.3. The group of displacements in the plane and Bessel's functions	46		
		2.II.4. The Bessel functions of purely imaginary argument	50		
		2.II.4.1. Integral representations of $I_{\nu}(z)$	51		
		2.II.5. The Hankel functions	56		
		2.II.5.1. Some integral representations for Hankel's functions	58		
		2.II.6. Addition formulae for Bessel's functions	62		
3.	THE	WAVE EQUATION	70		
	3.1.	Introduction			
	3.2.	The reflexion and refraction of a plane wave at the interface between two			
	W	homogeneous media			
	3.3.				
		Cylindrical waves			
	3.5.	Group velocity			
	3.6.	. Wave guide			

	3.7.	Successive reflexions of plane waves at two parallel rigid planes	82		
	3.8.	The relation between spherical and plane waves	84		
	3.9.	The reflexion of a spherical wave at a plane interface			
	3.10.	An alternative approach to Weyl's formula. Poritsky's generalisation	90		
4.	Asym	PTOTIC METHODS	97		
	4.1.	Asymptotic expansion	97		
	4.2.	. The asymptotic expansion of Hankel's functions in the neighborhood of the			
		point at infinity			
		The Laplace method	102		
		Asymptotic relations and Laplace's transform	103		
		The Laplace method (continued)	105		
		The method of steepest descent	108		
		Waves in linear dispersive media	111		
	4.8.	The asymptotic representation of the reflected wave in the problem of a spherical	12.2.2		
	4.0	wave impinging on a plane interface. The lateral wave	116		
	4.9.	The method of steepest descent; an extension to the case when some pole is located near the saddle	125		
	4.10		125 128		
		The asymptotic representation of Hankel's functions of large order An asymptotic representation of Legendre's functions of large order	134		
		The asymptotic representation of Hankel's functions of large order (continued)			
	4.12.	4.12.1. A discussion of the equation $\ddot{y} + 2\tau y = 0$	136		
		4.12.2. Approximate representations of $H_{\nu}^{(1)}(v)$, $dH_{\nu}^{(2)}(z)/dz _{z=\nu}$ for large order			
			139		
5	SCAT	TERING MATRIX THEORY	143		
•		Introduction	143		
		The direct problem	143		
	8 8 7	5.I.1. The one-dimensional model of wave propagation	143		
		5.I.2. An asymptotic approach	145		
		5.I.3. The matrix $S(k)$ (k real)	148		
		5.I.4. Evaluating the reflexion coefficient (k real)	150		
		5.I.5. The matrix $S(k)$ for k in the complex plane	152		
		5.I.6. The Fourier transform of $r(l, k)$, $r(k)$	157		
Pic.	5.II.	The inverse problem	160		
		5.II.1. Preliminary discussion	160		
		5.II.2. Some properties of $W_l(x, t)$, $W(x, t)$	163		
		5.II.3. The Gelfand-Levitan integral equation	174		
5.	FLOW	IN OPEN CHANNEL; ASYMPTOTIC SOLUTION OF SOME LINEAR AND NONLINEAR			
	WAVE	EQUATIONS	176		
		The kinematics and dynamics of flow in open channel	176		
		6.I.1. The geometric assumptions concerning a river bed	176		
		6.I.2. The dynamic assumptions	177		
		6.I.3. The basic equations of flow	177		
		6.I.4. An alternative approach to the problem	182		
		6.I.5. The pressure term	185		
		6.I.6. Long waves	189		
		6.I.7. Fourier transform of $th(kh_0)/kh_0$	189		
		6.I.8. The linearized equations of flow	191		
		6.I.9. The method of characteristics	192		
		6.I.10. Stability conditions	198		

.

20

		6.I.11. The analytic solution of the river flow equation	200				
	6.II.	The asymptotic representation of the solution of the wave equation	204				
	ää	6.II.1. An alternative approach	204				
		6.II.2. The asymptotic representation of the solution	209				
		6.II.3. The case $0 < c_2 < a < c_1$	223				
		6.II.4. The approximate treatment of the wave equation	227				
	6.III.		234				
	0,1111	6.III.1. The method of characteristics	234				
		6.III.2. The progressive wave	239				
		6.III.3. Non linear waves and the method of averaging	245				
		6.III.4. Non linear dispersive waves and two scale expansion procedure	253				
7.	Seisn	MIC WAVES	266				
	7.1.	Waves in elastic solids	266				
ē	7.2.	Plane waves	268				
	7.3.	Reflexion and refraction of plane elastic waves	269				
		Waves of kind I, II, III	272				
	7.5.	Analytic representation of P and S waves	276				
		The layered spherical model	280				
		The energy balance	282				
		The reflexion and transmission coefficients	286				
		The wave system in the layered spherical model	288				
		7.9.1. The case of an incident P wave	288				
		7.9.2. The case of an incident SV wave	290				
	7.10.	The P, PcP, PcS, PKP waves produced by a point source located outside the core	291				
	711	Application of the method of steepest descent to P and PcP wave integrals	295				
	7.11.	7.11.1. The P wave	299				
		7.11.2. The PcP wave	302				
	7.12.	2. The diffracted PcP wave					
	8. Sc	OME PROBLEMS IN WATER WAVE THEORY	311				
	Intro	oduction	311				
	8.I.	Oscillations in an infinite channel of variable depth	312				
		8.I.1. The shape of the channel	313				
		8.I.2. The representation of the solution in terms of Fourier integrals	316				
		8.I.3. The functional equations (8.25), (8.26)	317				
		8.I.4. Zeros and poles of $H(u)$. Uniqueness property. Functional					
		relations between $F(u)$ and $G(u)$	320				
		8.I.5. Determination of the paths C , Γ and relations between the					
		constants l, m, p, q	321				
		8.I.6. Asymptotic behavior of the solution	323				
		8.I.7. Discussion of the amplitudes; the reflexion coefficient	325				
	8.II.	A diffraction problem	328				
		8.II.1. The representation of the solution as a Laplace integral	330				
		8.II.2. A heuristic way for choosing the C and Γ contours	330				
		8.II.3. The boundary conditions	334				
		8.II.4. A solution to the functional equation (8.66)	337				
		8.II.5. The boundary conditions	340				
		8.II.6. Asymptotic behavior of the solution	341				
R	EFERE	NCES	345				
IN	IDEX		348				

	•	