CONTENTS

CHAPTER I *ANALYTIC FUNCTIONS* *1*

1.1 Integral Representations of Analytic Functions *1*
1.2 Series Representations of Analytic Functions *13*
1.3 Singularities at Isolated Points *17*
A. Isolated Singular Points in the Finite Plane *18*
B. Isolated Singularities at Infinity *24*
1.4 Entire and Meromorphic Functions; Theorem of Liouville *27*
1.5 The Lemma of Jordan *29*
1.6 Analytic Continuation *30*
A. Definition and Uniqueness of an Analytic Continuation *32*
B. Techniques of Analytic Continuation *35*
C. Illustrative Examples *39*
1.7 Multivalued Functions *43*
A. The Multivalued Function $\sqrt{\lambda}$ *44*
B. Multivalued Functions in Electrostatics *47*
C. Multivalued Functions in the Propagation of Harmonic Plane Waves *49*
D. The Multivalued Function $\log \lambda$ *54*
E. Evaluation of an Integral in Neutron Transport Theory *56*
F. Evaluation of the Inversion Integral of a Unilateral Fourier Transform *60*

CHAPTER II FOURIER TRANSFORMS, CAUSALITY, AND DISPERSION RELATIONS 64

2.0 Introduction *64*
2.1 Some Remarks on Integration and Cauchy Principal Value Integrals *69*
2.2 Infinite Integrals Depending on a Parameter *73*
2.3 Representation of an Analytic Function by an Infinite Integral *79*
2.4 Bilateral Fourier Transforms *83*
2.5 Some Elementary Properties of Bilateral Fourier Transforms *87*
2.6 Bilateral Fourier Transforms, Linear Physical Systems, and Green's Functions *94*
2.7 Small Oscillations of a Physical System about a Position of Stable Equilibrium *98*
2.8 Electric Polarization and Dispersion Relations *109*
2.9 Classical Motion of a Bound Electron in an Electric Field *111*
2.10 Unilateral Fourier Transforms *114*
2.11 Causal Fourier Transforms, Dispersion Relations, and Titchmarsh's Theorem *118*
2.12 Applications to Linear Physical Systems *134*
2.13 Analytic Continuation of a Function Defined by a Hilbert Transform Integral *136*
2.14 Asymptotic Representations of Unilateral Fourier Transforms *142*
A. Asymptotic Representations of Functions of a Real or Complex Variable *143*
B. Asymptotic Representations of Unilateral Fourier Transforms *148*
C. Asymptotic Behavior of an Inversion Integral for Large Values of t *151*
2.15 Applications to Integral Equations with a Displacement Kernel *153*
A. Homogeneous Integral Equations *155*
B. Inhomogeneous Integral Equations *159*

CHAPTER III THE WIENER-HOPF TECHNIQUE 162

3.0 Introduction *162*
3.1 The Mittag-Leffler Expansion *167*

3.2 The Decomposition of a Function that is Analytic in a Strip 169
3.3 Weierstrass's Factorization Formula 173
3.4 Wiener-Hopf Factorization Formulas 176
3.5 Wiener-Hopf Integral Equations of the Second Kind 184
3.6 Wiener-Hopf Integral Equations of the First Kind 190
3.7 Remarks on Some Partial Differential Equations of Mathematical Physics 198
3.8 Boundary Value Problems Involving the Helmholtz Equation 202
3.9 Small Amplitude Theory of Water Waves 207
3.10 Semi-Infinite Dock on a Sea of Finite Depth 210

CHAPTER IV BOUNDARY VALUE PROBLEMS FOR SECTIONALLY ANALYTIC FUNCTIONS 218

4.0 Introduction 218
4.1 Hölder Continuous Functions 220
4.2 Sectionally Continuous Functions 222
4.3 Sectionally Analytic Functions 225
4.4 Cauchy Integral Representations of Sectionally Analytic Functions 226
4.5 The Principal Value of a Cauchy Integral 228
4.6 The Formulas of Plemelj 231
4.7 Boundary Value Problems of Plemelj 235
4.8 Representation of Sectionally Analytic Functions by Cauchy Integrals Over an Infinite Line 236
4.9 The Poincaré-Bertrand Transformation Formula 242
4.10 Boundary Value Problems for Sectionally Analytic Functions in a Plane Cut Along a Contour 243
A. The Homogeneous Hilbert Problem for a Contour 244
B. The Inhomogeneous Hilbert Problem for a Contour 248
4.11 Singular Integral Equations of the Cauchy Type 250
4.12 Boundary Value Problems for Sectionally Analytic Functions in a Plane Cut Along a Finite Path 253
A. The Homogeneous Hilbert Problem for a Finite Path 253

B. The Inhomogeneous Hilbert Problem for a Finite Path 256
4.13 Boundary Value Problems for Sectionally Analytic Functions in a Plane Cut Along an Infinite Straight Line 258
A. The Homogeneous Hilbert Problem for an Infinite Line 258
B. The Inhomogeneous Hilbert Problem for an Infinite Line 262
4.14 Wiener-Hopf Integral Equations and Hilbert Problems 263
A. Homogeneous Wiener-Hopf Integral Equations of the Second Kind 263
B. Inhomogeneous Wiener-Hopf Integral Equations of the Second Kind 265
C. Wiener-Hopf Integral Equations of the First Kind 267
4.15 Green's Functions and the Solution of Boundary Value Problems in Electrostatics 269
4.16 Green's Function for a Point Charge on the Axis of an Infinite Cylinder 276
4.17 Green's Function for a Point Charge on the Axis of a Semi-Infinite Cylinder 279

CHAPTER V DISTRIBUTIONS 289

5.0 Introduction 289
5.1 Definition of a Schwartz Distribution 290
5.2 Algebraic and Analytic Operations on Distributions 299
5.3 The Derivative of a Distribution 306
5.4 Tempered Distributions and Their Fourier Transforms 313
5.5 The Cauchy Representation of Distributions 322
A. Cauchy Representations of Distributions on the Space E 323
B. Cauchy Representations of Distributions on the Space B 329
C. Cauchy Representations and Plemelj Formulas 335
5.6 Generalized Analytic Functions 339
5.7 Distributions and Boundary Values of Analytic Functions 349

CHAPTER VI APPLICATIONS IN NEUTRON TRANSPORT THEORY 361

6.0 Introduction *361*
6.1 Properties of Angular Eigenfunctions: Orthogonality *365*
6.2 Properties of Angular Eigenfunctions: Completeness *370*
6.3 Full Range Expansion Coefficients *377*
6.4 Half Range Expansion Coefficients *382*
6.5 Summary *386*
6.6 Some One-Dimensional Boundary Value Problems of Time-Independent Neutron Transport Theory *389*
A. The Infinite Medium Green's Function *390*
B. The Half Space Green's Function *394*
C. The Half Space Albedo Problem *398*
D. The Milne Problem *401*
E. Half Space With Constant Isotropic Source *405*
F. Bare Slab Criticality Problem *406*

CHAPTER VII APPLICATIONS IN PLASMA PHYSICS 411

7.0 Introduction *411*
7.1 The Collisionless Boltzmann Equation *413*
7.2 Longitudinal Oscillations in an Unbounded Plasma: Problem Statement *418*
7.3 Landau's Transform Solution of the Initial Value Problem *421*
7.4 The Time-Dependent Behavior of the Potential of the Self-Consistent Field *422*
7.5 Criteria for Growing or Damped Plasma Oscillations *436*
A. Jackson's Criterion for Bell-shaped Equilibrium Velocity Distributions *437*
B. Penrose's Criterion for Non-Maxwellian Equilibrium Velocity Distributions *439*
C. The Criterion of Nyquist *444*
D. McGune's Exact Inversion Procedure *448*
7.6 Long and Short Wave Length Plasma Oscillations *449*
A. Long Wave Length Plasma Oscillations *449*
B. Short Wave Length Plasma Oscillations *453*

7.7 Some Special Equilibrium Velocity Distributions 454
A. A Zero Temperature Plasma 454
B. The Two-Stream Velocity Distribution 456
C. The Lorentzian Equilibrium Velocity Distribution 457
D. The Maxwellian Equilibrium Velocity Distribution 458
7.8 Van Kampen's Normal Mode Solution of the Initial Value Problem for a Maxwellian Equilibrium Velocity Distribution 465
7.9 Case's Normal Mode Solution of the Initial Value Problem for General Equilibrium Velocity Distributions 472

APPENDIX A PATHS, CONTOURS, AND REGIONS IN THE COMPLEX PLANE 481

APPENDIX B ORDER RELATIONS 484

BIBLIOGRAPHY 487

INDEX 499