

Contents

<i>Preface</i>		xi
<i>Acknowledgments</i>		xiii
General Introduction and Outline		1
CHAPTER 1	Bäcklund Transformations and Their Application to Nonlinear Equations of Mathematical Physics	
1.1	The Origin and Importance of Bäcklund Transformations	12
1.2	The 1 + 1 Sine-Gordon Equation. The Permutability Theorem. Bianchi Diagrams	16
1.3	Ultrashort Optical Pulse Propagation in a Resonant Medium. $2N\pi$ Light Pulses	22
1.4	Propagation of Magnetic Flux through a Long Josephson Junction. An Elliptic Analog of the 1 + 1 Sine-Gordon Equation. Generation of Solutions via a Bäcklund Transformation	29
1.5	The Cole–Hopf Reduction of Burgers’ Equation to the Heat Equation. Solution of Initial Value Problems	38
1.6	Finite Amplitude Dispersive Waves and the Korteweg–deVries Equation	43
1.7	The Wahlquist–Estabrook Invariant Transformation of the Korteweg–deVries Equation. The Soliton Ladder	51

1.8	An Auto-Bäcklund Transformation of the Modified Korteweg–deVries Equation. The Permutability Theorem and Generation of Solutions	57
1.9	The Miura Transformation	60
1.10	A Nonlinear Schrödinger Equation	62
1.11	Bäcklund Transformations and the AKNS System. An Invariant Transformation of the Cubic Schrödinger Equation	65
1.12	The Hirota Bilinear Operator Formulation of Bäcklund Transformations. The Boussinesq Equation. Kaup's Higher Order Water Wave Equation	69
1.13	Bäcklund Transformations for Higher Order Korteweg–deVries Equations	79
1.14	Bäcklund Transformations in Higher Dimensions. The Sine-Gordon Equation in $3 + 1$ Dimensions. The Kadomtsev–Petviashvili Equation. The Yang Equations	82
1.15	The Benjamin–Ono Equation for Internal Deep Water Waves. The Nakamura Transformation	92
1.16	A Bäcklund Transformation for Joseph's Equation for Internal Waves in a Stratified Fluid with Finite Depth	101
1.17	Bäcklund Transformations of Nonlinear Lattice Equations in Their Continuum Approximation. The Konno–Sanuki Transformation	105
1.18	Bäcklund Transformations of Nonlinear Differential-Difference Equations. The Toda and Associated Lattices	110
1.19	Bäcklund Transformations in General Relativity. The Ernst Equations. Neugebauer's Permutability Theorem	115

CHAPTER 2 **A Local Jet-Bundle Formulation of Bäcklund Transformations**

2.1	Preliminaries	123
2.2	Contact Structures	126
2.3	Partial Differential Equations on Jet Bundles	129
2.4	Fibered Products of Jet Bundles	132
2.5	Bäcklund Maps	134
2.6	Bäcklund Transformations Determined by Bäcklund Maps	139
2.7	Symmetries of Differential Equations and One-Parameter Families of Bäcklund Maps	143
2.8	The Wahlquist–Estabrook Procedure	145

CHAPTER 3 **Bäcklund Transformations in Gasdynamics, Nonlinear Heat Conduction, and Magnetogasdynamics**

3.1	The Reciprocal Relations and the Haar Transformation	151
3.2	Properties of the Reciprocal Relations. Invariance of the Equation of State	155
3.3	Reciprocal Relations in Subsonic Gasdynamics	160
3.4	Reciprocal-Type Transformations in Steady Magnetogasdynamics	162

3.5	A Bäcklund Transformation in Nonlinear Heat Conduction	168
3.6	Bäcklund Transformations of the Loewner Type	171
3.7	Reduction of the Hodograph Equations to Canonical Form in Subsonic, Transsonic, and Supersonic Gasdynamics	176
3.8	Bäcklund Transformations in Aligned Nondissipative Magnetogasdynamics	183
3.9	Invariant Transformations in Nonsteady Gasdynamics	185
3.10	Bäcklund Transformations in Lagrangian Gasdynamics. Reflection of a Centered Wave in a Shock Tube	189

CHAPTER 4 **Bäcklund Transformations and Wave Propagation in Nonlinear Elastic and Nonlinear Dielectric Media**

4.1	Propagation of Large Amplitude Waves in Nonlinear Elastic Media. Reduction to Canonical Form of the Riemann Representation	197
4.2	The Model Constitutive Laws	202
4.3	Reflection and Transmission of a Large Amplitude Shockless Pulse at a Boundary	211
4.4	Electromagnetic Wave Propagation in Nonlinear Dielectric Media. Reduction to Canonical Form via Bäcklund Transformations	225
4.5	Evolution of a Large Amplitude Centered Fan in a Nonlinear Dielectric Slab	230

CHAPTER 5 **Bäcklund Transformations and Stress Distribution Theory in Elastostatics**

5.1	Weinstein's Correspondence Principle in the Context of Bäcklund Transformations of the Generalized Stokes–Beltrami System	238
5.2	Application of Weinstein's Correspondence Principle in Elastostatics. Associated Axially Symmetric Punch, Crack, and Torsion Boundary-Value Problems	242
5.3	Antiplane Crack and Contact Problems in Layered Elastic Media. Bäcklund Transformations and the Bergman Series Method	253
5.4	Stress Concentration for Shear-Strained Prismatic Bodies with a Nonlinear Stress–Strain Law	262

APPENDIX I **Properties of the Hirota Bilinear Operators** 274

APPENDIX II **Differential Forms**

II.1	Tangent Spaces and Vector Fields on \mathbb{R}^k	276
II.2	Differential p -Forms on \mathbb{R}^k	277
II.3	The Exterior Derivative	279

	II.4	Pull-Back Maps	282
	II.5	The Interior Product of Vector Fields and Differential p -Forms	283
	II.6	The Lie Derivative of Differential Forms	284
APPENDIX III		Differential Forms on Jet Bundles	
	III.1	Preliminaries	286
	III.2	Exterior Differential Systems on Jet Bundles	288
	III.3	The System of Differential Equations Associated with an Exterior Differential System	289
	III.4	An Exterior Differential System of m -Forms on $J^k(M, N)$ Associated with a Quasi-Linear Equation on $J^{k+1}(M, N)$	290
APPENDIX IV		The Derivation of the Equations that Define $B^{k+1,1}(\psi)$	292
APPENDIX V		Symmetries of Differential Equations and Exterior Systems	
	V.1	Point Transformations	294
	V.2	Symmetries	297
APPENDIX VI		Composition of Symmetries and Bäcklund Maps	299
		References	302
		<i>Author Index</i>	323
		<i>Subject Index</i>	329

