

CONTENTS

Difference Methods for Initial-Value Problems

PART I: GENERAL CONSIDERATIONS

CHAPTER I

Introduction

1. Initial-value problems	3
2. The heat-flow problem	4
3. Finite-difference equations	6
4. Stability	9
5. Implicit difference equations	16
6. The truncation error	18
7. Rate of convergence	21
8. Comments on high-order formulas and rounding errors	24
9. Outline of remainder of book	25

CHAPTER II

Linear Operators

1. The function space of an initial-value problem	28
2. Banach spaces	31
3. Linear operators in a Banach space	33
4. The extension theorem	34
5. The principle of uniform boundedness	35

CHAPTER III

Linear Difference Equations

1. Properly posed initial-value problems	38
2. Finite-difference approximations	40
3. Convergence	43
4. Stability	43
5. Lax's equivalence theorem	44

CHAPTER IV

Pure Initial-Value Problems with Constant Coefficients

1. The class of problems	49
2. Fourier series	50
3. The Banach spaces	50
4. Properly posed initial-value problems	52
5. The finite-difference equations	53
6. The consistency condition	55
7. Stability	58
8. The von Neumann condition	59
9. First sufficient condition	61
10. Second sufficient condition	62
11. Third sufficient condition	64
12. Fourth sufficient condition	67
13. Conclusions	71

CHAPTER V

Multi-Level Difference Equations

1. Notation	74
2. Auxiliary Banach space	75
3. The equivalence theorem	77
4. Consistency	81
5. Example of Dufort and Frankel	83
6. Summary	85

PART II: APPLICATIONS

CHAPTER VI

Diffusion and Heat Flow

1. Examples of diffusion	89
2. The simplest heat-flow problem	91
3. Variable coefficients	96
4. Effect of lower order terms on stability	98
5. Solution of the implicit equations	101
6. A non-linear problem	104
7. Some results of Fritz John	109
8. Problems in several space variables	112

CHAPTER VII

The Transport Equation

1. Physical basis	121
2. The general neutron transport equation	122
3. Homogeneous slab: one group	125
4. Homogeneous sphere: one group	126
5. The "spherical harmonic" method	127
6. Slab: difference system I	131
7. A paradox	134
8. Slab: difference system II (Friedrichs)	135
9. Implicit schemes	137
10. The Wick-Chandrasekhar method for the slab	137
11. Equivalence of the two methods	139
12. Boundary conditions	141
13. Difference systems I and II	142
14. System III: forward and backward space differences	142
15. System IV (implicit)	144
16. System V (Carlson's scheme)	144
17. Generalization of the Wick-Chandrasekhar method	147
18. The S_n method of Carlson (1953)	148
19. A direct integration method	151

CHAPTER VIII

Sound Waves

1. Physical basis	165
2. The usual finite-difference equation	166
3. An implicit system	169
4. Coupled sound and heat-flow	170

CHAPTER IX

Elastic Vibrations

1. Vibrations of a thin beam	173
2. Explicit difference equations	175
3. An implicit system	177
4. Virtue of the implicit system	177
5. Solution of implicit equations of arbitrary order	178
6. Vibration of a bar under tension	185

CHAPTER X**Fluid Dynamics in One Space Variable**

1. Introduction	189
2. The Eulerian equations	191
3. Difference equations (Eulerian)	193
4. Stability of the difference equations (Eulerian)	195
5. The Lagrangean equations	198
6. Difference equations (Lagrangean)	200
7. Treatment of interfaces	202
8. The jump conditions at a shock	205
9. Effect of dissipation	208
10. Finite-difference equations	215
11. Stability of the finite-difference equations	218
12. Numerical tests of the pseudo-viscosity method	222
References	230
Index	234

