

Contents

Part I

GENERAL CONSIDERATIONS

1 Introduction

1.1	Initial-Value Problems	3
1.2	The Heat Flow Problem	4
1.3	Finite-Difference Equations	7
1.4	Stability	9
1.5	Implicit Difference Equations	16
1.6	The Truncation Error	19
1.7	Rate of Convergence	22
1.8	Comments on High-Order Formulas and Rounding Errors	24
1.9	Outline of the Remainder of the Book	25

2 Linear Operators

2.1	The Function Space of an Initial-Value Problem	28
2.2	Banach Spaces	30
2.3	Linear Operators in a Banach Space	33
2.4	The Extension Theorem	34
2.5	The Principle of Uniform Boundedness	34
2.6	A Fundamental Convergence Theorem	37
2.7	Closed Operators	37

3 Linear Difference Equations

3.1	Properly Posed Initial-Value Problems	39
3.2	Finite-Difference Approximations	42
3.3	Convergence	44
3.4	Stability	45
3.5	Lax's Equivalence Theorem	45
3.6	The Closed Operator A'	49
3.7	Inhomogeneous Problems	52
3.8	Change of Norm	57
3.9	Stability and Perturbations	58

4 Pure Initial-Value Problems with Constant Coefficients

4.1	The Class of Problems	60
4.2	Fourier Series and Integrals	61

4.3	Properly Posed Initial-Value Problems	63
4.4	The Finite-Difference Equations	65
4.5	Order of Accuracy and the Consistency Condition	67
4.6	Stability	68
4.7	The von Neumann Condition	70
4.8	A Simple Sufficient Condition	72
4.9	The Kreiss Matrix Theorem	73
4.10	The Buchanan Stability Criterion	80
4.11	Further Sufficient Conditions for Stability	83
5	Linear Problems with Variable Coefficients; Non-Linear Problems	
5.1	Introduction	91
5.2	Alternative Definitions of Stability	95
5.3	Parabolic Equations.	100
5.4	Dissipative Difference Schemes for Symmetric Hyperbolic Equations	108
5.5	Further Results for Symmetric Hyperbolic Equations	119
5.6	Non-Linear Equations with Smooth Solutions	124
6	Mixed Initial-Boundary-Value Problems	
6.1	Introduction	131
6.2	Basic Ideas of the Energy Method.	132
6.3	Simple Examples of the Energy Method: Stable Choice of Approxima- tions to Boundary Conditions and to Non-Linear Terms	137
6.4	Coupled Sound and Heat Flow	143
6.5	Mixed Problems for Symmetric Hyperbolic Systems	146
6.6	Normal Mode Analysis and the Godunov-Ryabenkii Stability Criterion	151
6.7	Application of the G-R Criterion to Mixed Problems	156
6.8	Conclusions	164
7	Multi-Level Difference Equations	
7.1	Notation	168
7.2	Auxiliary Banach Space	169
7.3	The Equivalence Theorem	171
7.4	Consistency and Order of Accuracy	174
7.5	Example of Du Fort and Frankel	176
7.6	Summary	179

Part II

APPLICATIONS

Preface to Part II	183
------------------------------	-----

8 Diffusion and Heat Flow

8.1	Examples of Diffusion	185
8.2	The Simplest Heat-Flow Problem	186
8.3	Variable Coefficients.	193
8.4	Effect of Lower Order Terms on Stability	195
8.5	Solution of the Implicit Equations	198
8.6	A Non-Linear Problem	201
8.7	Problems in Several Space Variables	206
8.8	Alternating-Direction Methods.	211
8.9	Splitting and Fractional-Step Methods	216

9 The Transport Equation

9.1	Physical Basis	218
9.2	The General Neutron Transport Equation	219
9.3	Homogeneous Slab: One Group	222
9.4	Homogeneous Sphere: One Group	223
9.5	The "Spherical Harmonic" Method	224
9.6	Slab: Difference System I for Hyperbolic Equations	228
9.7	A Paradox	230
9.8	Slab: Difference System II (Friedrichs)	232
9.9	Implicit Schemes.	233
9.10	The Wick-Chandrasekhar Method for the Slab	233
9.11	Equivalence of the Two Methods	235
9.12	Boundary Conditions	237
9.13	Difference Systems I and II	237
9.14	System III: Forward and Backward Space Differences	238
9.15	System IV (Implicit)	239
9.16	System V (Carlson's Scheme)	240
9.17	Generalization of the Wick-Chandrasekhar Method	243
9.18	The S_n Method of Carlson (1953)	244
9.19	A Direct Integration Method	246

10 Sound Waves

10.1	Physical Basis	259
10.2	The Usual Finite-Difference Equation	260
10.3	An Implicit System	263
10.4	Coupled Sound and Heat Flow	264
10.5	A Practical Stability Criterion	269

11 Elastic Vibrations

11.1	Vibrations of a Thin Beam	271
11.2	Explicit Difference Equations	273

11.3	An Implicit System	274
11.4	Virtue of the Implicit System	275
11.5	Solution of Implicit Equations of Arbitrary Order	275
11.6	Vibration of a Bar Under Tension	282
12	Fluid Dynamics in One Space Variable	
12.1	Introduction	288
12.2	The Eulerian Equations	289
12.3	Difference Equations, Eulerian	290
12.4	The Lagrangean Equations	293
12.5	Difference Equations, Lagrangean	295
12.6	Treatment of Interfaces in the Lagrangean Formulation	298
12.7	Conservation-Law Form and the Lax-Wendroff Equations	300
12.8	The Jump Conditions at a Shock	306
12.9	Shock Fitting	308
12.10	Effect of Dissipation	311
12.11	Finite-Difference Equations	317
12.12	Stability of the Finite-Difference Equations	320
12.13	Numerical Tests of the Pseudo-Viscosity Method	324
12.14	The Lax-Wendroff Treatment of Shocks	330
12.15	The Method of S. K. Godunov	338
12.16	Magneto-Fluid Dynamics	345
13	Multi-Dimensional Fluid Dynamics	
13.1	Introduction	351
13.2	The Multi-Dimensional Fluid-Dynamic Equations	354
13.3	Properly and Improperly Posed Problems	356
13.4	The Two-Step Lax-Wendroff or L-W Method	360
13.5	The Viscosity Term for the L-W Method	365
13.6	Piecewise Analytic Initial-Value Problems	368
13.7	A Program for the Development of Methods	374
13.8	Characteristics in Two-Dimensional Flow	375
13.9	Shock Fitting in Two Dimensions	378
13.10	The Problem of the Atmospheric Front	383
	References	389
	Subject Index	401

