

CONTENTS

	1.1.2 Notation, 7
	1.1.3 Integral Relations, 9
	1.1.4 Classification of Equations, 10
	Exercises 1.1, 10
1.2	KINETICS, 14
	1.2.1 The Stress at a Point, 14
	1.2.2 Stress Tensor and the Cauchy Formula, 16
	1.2.3 Transformations of Stress Components, 19
	1.2.4 Principal Stresses and Principal Planes, 22
	1.2.5 Equations of Motion, 25
	1.2.6 Symmetry of the Stress Tensor, 28
	Exercises 1.2, 30
1.3	KINEMATICS, 32
	1.3.1 Strain at a Point, 32
	1.3.2 Transformation of Strain Components, 35
	1.3.3 Strain Compatibility Equations, 38
	Exercises 1.3, 41

1.4 THERMODYNAMIC PRINCIPLES, 42

1.4.1 Introduction, 42

1 A REVIEW OF THE EQUATIONS OF MECHANICS

1.1.1 Configuration and Coordinates, 6

1.1 Introduction, 1

1.5

		The First Law of Thermodynamics: Energy Equation, 43 The Second Law of Thermodynamics, 45			
	Exerc	CISES 1.4, 45			
1.5	Constitutive Equations, 46				
	1.5.2 1.5.3 1.5.4 1.5.5	Generalized Hooke's Law, 46 Strain Energy Density Function, 47 Elastic Symmetry, 49 Transformation of Elastic Moduli, 54 Thermoelastic Constitutive Equations, 56 EISES 1.5, 56			
1.6	Boun	DARY-VALUE PROBLEMS OF MECHANICS, 59			
	1.6.2 1.6.3	Summary of Equations, 59 Initial and Boundary Conditions, 61 Classification of Boundary-Value Problems, 61 Existence and Uniqueness of Solutions, 63			
	Exerc	CISES 1.6, 65			
1.7	Equa	tions of Bars, Beams, Torsion, and Plane Elasticity, 67			
	1.7.2 1.7.3 1.7.4	Introduction, 67 Axially Loaded Members: Bars, 68 Theory of Straight Beams, 70 Torsion of Prismatic Members, 78 Plane Elasticity, 81			
	Exerc	CISES 1.7, 84			
2]	ENER(GY AND VARIATIONAL PRINCIPLES	89		
2.1	Preliminary Concepts, 89				
	2.1.2 2.1.3	Introduction, 89 Work and Energy, 90 Strain Energy and Complementary Strain Energy, 92 Virtual Work, 97			
	Exer	CISES 2.1, 101			
2.2	CALC	ulus of Variations, 102			
	2.2.1	The Variational Operator, 102			

2.2.2 The First Variation of a Functional, 105

Contents

Z.Z. Z.	2.2.3	Extremum	of	a	Functional.	100
--	-------	----------	----	---	-------------	-----

- 2.2.4 The Euler Equations, 107
- 2.2.5 Natural and Essential Boundary Conditions, 108
- 2.2.6 A More General Functional, 110
- 2.2.7 Minimization with Linear Equality Constraints, 114

EXERCISES 2.2, 119

2.3 VIRTUAL WORK AND ENERGY PRINCIPLES, 121

- 2.3.1 Introduction, 121
- 2.3.2 Principle of Virtual Displacements, 121
- 2.3.3 Unit-Dummy-Displacement Method, 126
- 2.3.4 Principle of Total Potential Energy, 129
- 2.3.5 Principles of Virtual Forces and Complementary Potential Energy, 132
- 2.3.6 Unit-Dummy-Load Method, 135

EXERCISES 2.3, 136

2.4 STATIONARY VARIATIONAL PRINCIPLES, 139

- 2.4.1 Introduction, 139
- 2.4.2 The Hellinger-Reissner Variational Principle, 140
- 2.4.3 The Reissner Variational Principle, 142

EXERCISES 2.4, 144

2.5 Hamilton's Principle, 146

- 2.5.1 Introduction, 146
- 2.5.2 Hamilton's Principle for Particles and Rigid Bodies, 146
- 2.5.3 Hamilton's Principle for a Continuum, 151
- 2.5.4 Hamilton's Principle for Constrained Systems, 155

EXERCISES 2.5, 158

2.6 Energy Theorems of Structural Mechanics, 160

- 2.6.1 Introduction, 160
- 2.6.2 Castigliano's First Theorem, 161
- 2.6.3 Castigliano's Second Theorem, 166
- 2.6.4 Betti's and Maxwell's Reciprocity Theorems, 170

EXERCISES 2.6, 175

3 VARIATIONAL METHODS OF APPROXIMATION

177

3.1 SOME PRELIMINARIES, 177

		Introduction, 177 Some Mathematical Concepts, 178				
	Exerc	CISES 3.1, 182				
3.2	THE F	THE RITZ METHOD, 183				
	3.2.2 3.2.4 3.2.5	Introduction, 183 Description of the Method, 184 Matrix Form of the Ritz Equations, 187 Illustrative Examples, 188 Application to General Boundary-Value Problems, 203				
	Exerc	CISES 3.2, 210				
3.3	WEIG	HTED-RESIDUAL METHODS, 211				
	3.3.2 3.3.4 3.3.5	Introduction, 211 The Galerkin Method, 217 Least-Squares, Collocation, and Subdomain Methods, 221 The Kantorovich Method, 229 The Trefftz Method, 239 Closing Remarks on Traditional Variational Methods, 243				
	Exerc	CISES 3.3, 244				
3.4	THE F	FINITE ELEMENT METHOD, 247				
	3.4.2 3.4.3 3.4.4	Introduction, 247 One-Dimensional Second-Order Equations, 248 One-Dimensional Fourth-Order Equations, 267 Two-Dimensional Second-Order Equations, 276 Closure, 301				
	Exerc	CISES 3.4, 302				
4	THEO	RY AND ANALYSIS OF PLATES AND SHELLS	310			
4.1	CLASSICAL THEORY OF PLATES, 310					
	4.1.2	Governing Equations, 310 Exact Solutions, 324 Variational Solutions, 336				
	Exer	CISES 4.1, 347				
4.2	SHEAR	R DEFORMATION THEORIES OF PLATES, 354				

4.2.1 Introduction, 354

Contents	xiii
Committee	

		A First-Order Theory, 356 A Refined Higher-Order Theory, 364		
	28	CISES 4.2, 384		
4.3	Lamin	NATED COMPOSITE PLATES, 389		
	4.3.2	Introduction, 389 A First-Order Theory, 390 A Refined Higher-Order Theory, 401		
	Exer	CISES 4.3, 406		
4.4	THEO	RY OF SHELLS, 408		
	4.4.2 4.4.3 4.4.4	Introduction, 408 Geometric Relations, 410 Stress Resultants, 412 Equations of Motion, 413 Exact Solutions, 417		
	Exerc	CISES 4.4, 426		
4.5	FINIT	e-Element Analysis of Plates and Shells, 427		
	4.5.2 4.5.3 4.5.4	Introduction, 427 Displacement Models of the Classical Plate Theory, 428 Mixed Models of the Classical Plate Theory, 433 Shear Deformable Plate Elements, 449 Shear Deformable Shell Elements, 463		
	Exerc	CISES 4.5, 469		
BIB	LIOGF	RAPHY	474	
ANS	SWERS	S TO SELECTED EXERCISES	496	
IND	NDEX			