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**Atomic and Molecular Physics Experiments in Quantum  
Chaology**  
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**Topics in Quantum Chaos**  
*R E Prange*

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