



## TABLE OF CONTENTS

|                                                                                               |    |
|-----------------------------------------------------------------------------------------------|----|
| Preface                                                                                       | ix |
| <b><u>PART I - DYNAMICAL SYSTEMS AND INVERSE SCATTERING PROBLEMS</u></b>                      |    |
| INTEGRABLE MANY-BODY PROBLEMS / F.Calogero                                                    | 3  |
| 1. Introduction                                                                               | 3  |
| 2. Many-Body Problems Solvable by the Lax Trick                                               | 3  |
| 3. Motion of Poles of Nonlinear Partial Differential Equations and Related Many-Body Problems | 27 |
| 4. Motion of Zeros of Linear Evolution Equations and Related Integrable Many-Body Problems    | 40 |
| INVERSE SCATTERING PROBLEMS FOR NONLINEAR APPLICATIONS / P.C.Sabatier                         | 55 |
| 1. Introduction                                                                               | 55 |
| 2. Scattering Problems before Nonlinear Applications                                          | 56 |
| 3. Structure of the I.S.P. Solution : The Transformation Operator                             | 63 |
| 4. Structure of the I.S.P. Solution : The Integral Equation                                   | 66 |
| 5. Construction of Nonlinear Equations                                                        | 76 |
| 6. A Concluding Diagram                                                                       | 78 |
| SOLUTIONS OF NONLINEAR EQUATIONS SIMULATING PAIR PRODUCTION AND PAIR ANNIHILATION / A.O.Barut | 81 |
| THE TWO-TIME METHOD APPLIED TO SLOWLY EVOLVING OSCILLATING SYSTEMS / J.Robert Buchler         | 85 |
| 1. Introduction                                                                               | 85 |
| 2. The Two-Time Method                                                                        | 88 |
| 3. The Harmonic Approximation and Vibrational Stability Analysis                              | 91 |
| 4. Summary                                                                                    | 95 |

**PART II - SOLITONS**

|                                                                                                              |                |
|--------------------------------------------------------------------------------------------------------------|----------------|
| <b>SOLITONS IN PHYSICS / R.K.Bullough</b>                                                                    | <b>99</b>      |
| 1. Introductory Mathematics                                                                                  | 99             |
| 2. Applications of Solitons to Nonlinear Physics                                                             | 114            |
| 3. Some Particular Applications of Solitons in Physics                                                       | 117            |
| 4. Quantized Solitons                                                                                        | 133            |
| <br><b>SOLITONS AND GEOMETRY / Fernando Lund</b>                                                             | <br><b>143</b> |
| 1. Introduction                                                                                              | 143            |
| 2. Geometry                                                                                                  | 145            |
| 3. Physics                                                                                                   | 153            |
| 4. Solitons                                                                                                  | 163            |
| <br><b>HIROTA'S METHOD OF SOLVING SOLITON-TYPE EQUATIONS / P.J.Caudrey</b>                                   | <br><b>177</b> |
| 1. The Korteweg-de Vries Equation                                                                            | 177            |
| 2. The Sine-Gordon Equation                                                                                  | 181            |
| 3. The Double-Sine-Gordon Equation                                                                           | 183            |
| 4. A Hierarchy of KdV Equations                                                                              | 185            |
| 5. Polynomial Conserved Densities                                                                            | 190            |
| <br><b>PROLONGATION STRUCTURE TECHNIQUES FOR THE NEW HIERARCHY OF KORTEWEG-DE VRIES EQUATIONS / R.K.Dodd</b> | <br><b>193</b> |
| <br><b>PERTURBATION THEORY FOR THE DOUBLE SINE-GORDON EQUATION / A.L.Mason</b>                               | <br><b>205</b> |

**PART III - DISCRETE SYSTEMS AND CONTINUUM MECHANICS**

|                                                                                                                 |                |
|-----------------------------------------------------------------------------------------------------------------|----------------|
| <b>PAINLEVÉ TRANSCENDENTS AND SCALING FUNCTIONS OF THE TWO-DIMENSIONAL ISING MODEL / Craig A.Tracy</b>          | <b>221</b>     |
| 1. Introduction                                                                                                 | 221            |
| 2. Two-Dimensional Ising Model                                                                                  | 222            |
| 3. Scaling Limit and Scaling Functions                                                                          | 223            |
| 4. Explicit Formulas for $\hat{F}_{\pm}(x)$                                                                     | 224            |
| 5. Painlevé Transcendents                                                                                       | 228            |
| <br><b>STATISTICAL MECHANICS OF NONLINEAR LATTICE DYNAMIC MODELS EXHIBITING PHASE TRANSITIONS / T.Schneider</b> | <br><b>239</b> |
| 1. Introduction                                                                                                 | 239            |
| 2. The Models and their Continuum Limit, the Equations of Motion and Particular Solutions                       | 241            |
| 3. Dynamic Variables, Conservation Laws and Spectral Densities                                                  | 247            |
| 4. Molecular-Dynamics Technique                                                                                 | 249            |
| 5. Molecular-Dynamics Results; Static and Dynamic Equilibrium Properties                                        | 251            |
| 6. Nonlinear Heat-Pulse Propagation                                                                             | 260            |

|                                                                      |     |
|----------------------------------------------------------------------|-----|
| NONLOCAL CONTINUUM MECHANICS AND SOME APPLICATIONS / A.Cemal Eringen | 271 |
| 1. Introduction                                                      | 271 |
| 2. Balance Laws                                                      | 274 |
| 3. Constitutive Equations                                            | 276 |
| 4. Thermodynamic Restrictions                                        | 279 |
| 5. Linear Theory                                                     | 281 |
| 6. Determination of Nonlocal Elastic Moduli                          | 285 |
| 7. Surface Waves                                                     | 289 |
| 8. Screw Dislocation                                                 | 291 |
| 9. Fracture Mechanics                                                | 295 |
| 10. Nonlocal Fluid Mechanics and Turbulence                          | 303 |

#### PART IV - NONLINEAR FIELD THEORIES AND QUANTIZATION

|                                                        |     |
|--------------------------------------------------------|-----|
| QUANTIZATION OF A NONLINEAR FIELD EQUATION / L.Castell | 321 |
| 1. The Classical Theory                                | 322 |
| 2. Symmetry Transformations                            | 324 |
| 3. Quantum Mechanics of the Free Field Equation        | 324 |
| 4. Compactification of Time                            | 325 |
| 5. Perturbation Expansion                              | 326 |
| 6. The Classical Scattering Theory                     | 327 |
| 7. The Quantization                                    | 328 |
| 8. The Commutation Relations                           | 332 |
| 9. Quantization in Analogy to the Thirring Model       | 332 |

|                                                                  |     |
|------------------------------------------------------------------|-----|
| CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD EQUATIONS / A.O.Barut | 335 |
| 1. Linear Fields                                                 | 335 |
| 2. Nonlinear Fields                                              | 337 |
| 3. Multicomponent Fields                                         | 341 |
| 4. Nonlinear Chiral Fields                                       | 341 |
| 5. Physical Interpretation and Applications                      | 343 |
| 6. Choice of Nonlinear Model                                     | 344 |

|                                                                                                                    |     |
|--------------------------------------------------------------------------------------------------------------------|-----|
| NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo | 347 |
| 1. A General Field Theoretical Problem                                                                             | 347 |
| 2. An Application of the Canonical Formalism                                                                       | 349 |

|                                                                 |     |
|-----------------------------------------------------------------|-----|
| NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert | 355 |
| 1. Introduction                                                 | 355 |
| 2. A Simple Model                                               | 356 |
| 3. Presence of Pairing Force                                    | 366 |
| 4. Conclusion                                                   | 373 |

|                                                                                       |         |
|---------------------------------------------------------------------------------------|---------|
| NONPERTURBATIVE SELF-INTERACTIONS, SOLITARY WAVES<br>AND OTHERS / Philip B.Burt       | 375     |
| 1. Introduction                                                                       | 375     |
| 2. Nonperturbative, Self-Interacting Quantum Fields                                   | 376     |
| 3. Solitary Wave Propagators                                                          | 381     |
| 4. Solitary Waves and Others                                                          | 387     |
| 5. Concluding Remarks                                                                 | 396     |
| <br>BOUND STATES OF FERMIONS IN EXTERNAL AND<br>SELF-CONSISTENT FIELDS / J.Rafelski   | <br>399 |
| 1. Solutions of the Dirac Equation                                                    | 400     |
| 2. Quantum Field Theory of Spin- $\frac{1}{2}$ Particles in<br>Strong External Fields | 414     |
| 3. Supercharged Vacuum and Klein's Paradox                                            | 424     |
| 4. Strong Fields in Quantum Field Theory                                              | 434     |
| <br>Subject Index                                                                     | <br>469 |

