



# Contents

<b>Preface</b>	<b>V</b>
<b>Foreword : People and events behind the “Science of Fractal Images”</b>	<b>1</b>
<i>Benoit B.Mandelbrot</i>	
0.1 The prehistory of some fractals-to-be: Poincaré, Fricke, Klein and Escher	2
0.2 Fractals at IBM . . . . .	5
0.3 The fractal mountains by R.F. Voss . . . . .	6
0.4 Old films . . . . .	8
0.5 Star Trek II	8
0.6 Midpoint displacement in Greek geometry: The Archimedes construction for the parabola . . . . .	11
0.7 Fractal clouds . . . . .	12
0.8 Fractal trees . . . . .	13
0.9 Iteration, yesterday’s dry mathematics and today’s weird and wonderful new fractal shapes, and the “Geometry Supercomputer Project” . . . . .	14
0.10 Devaney, Barnsley and the Bremen “Beauty of Fractals” . . . .	17
<b>1 Fractals in nature: From characterization to simulation</b>	<b>21</b>
<i>Richard F.Voss</i>	
1.1 Visual introduction to fractals: Coastlines, mountains and clouds	22
1.1.1 Mathematical monsters: The fractal heritage . . . . .	25
1.1.2 Fractals and self-similarity . . . . .	25
1.1.3 An early monster: The von Koch snowflake curve . . .	26
1.1.4 Self-similarity and dimension . . . . .	28
1.1.5 Statistical self-similarity . . . . .	30
1.1.6 Mandelbrot landscapes . . . . .	30
1.1.7 Fractally distributed craters . . . . .	31
1.1.8 Fractal planet: Brownian motion on a sphere . . . . .	33
1.1.9 Fractal flakes and clouds . . . . .	33
1.2 Fractals in nature: A brief survey from aggregation to music . .	35
1.2.1 Fractals at large scales . . . . .	36
1.2.2 Fractals at small scales: Condensing matter . . . . .	36

1.2.3	Scaling randomness in time: $\frac{1}{f^\beta}$ -noises . . . . .	39
1.2.4	Fractal music . . . . .	40
1.3	Mathematical models: Fractional Brownian motion . . . . .	42
1.3.1	Self-affinity . . . . .	44
1.3.2	Zerosets . . . . .	45
1.3.3	Self-affinity in higher dimensions : Mandelbrot landscapes and clouds . . . . .	45
1.3.4	Spectral densities for fBm and the spectral exponent $\beta$ . . . . .	47
1.4	Algorithms: Approximating fBm on a finite grid . . . . .	47
1.4.1	Brownian motion as independent cuts . . . . .	48
1.4.2	Fast Fourier Transform filtering . . . . .	49
1.4.3	Random midpoint displacement . . . . .	51
1.4.4	Successive random additions . . . . .	54
1.4.5	Weierstrass-Mandelbrot random fractal function . . . . .	55
1.5	Laputa: A concluding tale . . . . .	57
1.6	Mathematical details and formalism . . . . .	58
1.6.1	Fractional Brownian motion . . . . .	58
1.6.2	Exact and statistical self-similarity . . . . .	59
1.6.3	Measuring the fractal dimension $D$ . . . . .	61
1.6.4	Self-affinity . . . . .	62
1.6.5	The relation of $D$ to $H$ for self-affine fractional Brownian motion . . . . .	63
1.6.6	Trails of fBm . . . . .	64
1.6.7	Self-affinity in $E$ dimensions . . . . .	64
1.6.8	Spectral densities for fBm and the spectral exponent $\beta$ . . . . .	65
1.6.9	Measuring fractal dimensions: Mandelbrot measures . . . . .	66
1.6.10	Lacunarity . . . . .	67
1.6.11	Random cuts with $H \neq \frac{1}{2}$ : Campbell's theorem . . . . .	69
1.6.12	FFT filtering in 2 and 3 dimensions . . . . .	69
<b>2</b>	<b>Algorithms for random fractals</b> . . . . .	<b>71</b>
	<i>Dietmar Saupe</i>	
2.1	Introduction . . . . .	71
2.2	First case study: One-dimensional Brownian motion . . . . .	74
2.2.1	Definitions . . . . .	74
2.2.2	Integrating white noise . . . . .	75
2.2.3	Generating Gaussian random numbers . . . . .	76
2.2.4	Random midpoint displacement method . . . . .	78
2.2.5	Independent jumps . . . . .	80
2.3	Fractional Brownian motion : Approximation by spatial methods . . . . .	82
2.3.1	Definitions . . . . .	82
2.3.2	Midpoint displacement methods . . . . .	84
2.3.3	Displacing interpolated points . . . . .	87

2.4	Fractional Brownian motion : Approximation by spectral synthesis . . . . .	90
2.4.1	The spectral representation of random functions . . . . .	90
2.4.2	The spectral exponent $\beta$ in fractional Brownian motion . . . . .	91
2.4.3	The Fourier filtering method . . . . .	93
2.5	Extensions to higher dimensions . . . . .	95
2.5.1	Definitions . . . . .	95
2.5.2	Displacement methods . . . . .	96
2.5.3	The Fourier filtering method . . . . .	105
2.6	Generalized stochastic subdivision and spectral synthesis of ocean waves . . . . .	109
2.7	Computer graphics for smooth and fractal surfaces . . . . .	112
2.7.1	Top view with color mapped elevations . . . . .	112
2.7.2	Extended floating horizon method . . . . .	113
	<b>Color plates and captions</b> . . . . .	<b>114</b>
2.7.3	The data and the projection . . . . .	126
2.7.4	A simple illumination model . . . . .	127
2.7.5	The rendering . . . . .	128
2.7.6	Data manipulation . . . . .	130
2.7.7	Color, anti-aliasing and shadows . . . . .	130
2.7.8	Data storage considerations . . . . .	131
2.8	Random variables and random functions . . . . .	133
<b>3</b>	<b>Fractal patterns arising in chaotic dynamical systems</b> . . . . .	<b>137</b>
	<i>Robert L.Devaney</i>	
3.1	Introduction . . . . .	137
3.1.1	Dynamical systems . . . . .	138
3.1.2	An example from ecology . . . . .	139
3.1.3	Iteration . . . . .	141
3.1.4	Orbits . . . . .	143
3.2	Chaotic dynamical systems . . . . .	145
3.2.1	Instability: The chaotic set . . . . .	145
3.2.2	A chaotic set in the plane . . . . .	146
3.2.3	A chaotic gingerbreadman . . . . .	149
3.3	Complex dynamical systems . . . . .	150
3.3.1	Complex maps . . . . .	150
3.3.2	The Julia set . . . . .	152
3.3.3	Julia sets as basin boundaries . . . . .	154
3.3.4	Other Julia sets . . . . .	155
3.3.5	Exploding Julia sets . . . . .	159
3.3.6	Intermittency . . . . .	163

<b>4</b>	<b>Fantastic deterministic fractals</b>	<b>169</b>
	<i>Heinz-Otto Peitgen</i>	
4.1	Introduction . . . . .	169
4.2	The quadratic family . . . . .	170
4.2.1	The Mandelbrot set . . . . .	177
4.2.2	Hunting for $K_c$ in the plane - the role of critical points .	180
4.2.3	Level sets	182
4.2.4	Equipotential curves . . . . .	183
4.2.5	Distance estimators . . . . .	192
4.2.6	External angles and binary decompositions . . . . .	192
4.2.7	Mandelbrot set as one-page-dictionary of Julia sets . .	199
4.3	Generalizations and extensions . . . . .	207
4.3.1	Newton's Method . . . . .	207
4.3.2	Sullivan classification . . . . .	210
4.3.3	The quadratic family revisited . . . . .	210
4.3.4	Polynomials . . . . .	212
4.3.5	A special map of degree four . . . . .	212
4.3.6	Newton's method for real equations . . . . .	213
4.3.7	Special effects . . . . .	214
<b>5</b>	<b>Fractal modelling of real world images</b>	<b>219</b>
	<i>Michael F. Barnsley</i>	
5.1	Introduction . . . . .	219
5.2	Background references and introductory comments . . . . .	221
5.3	Intuitive introduction to IFS: Chaos and measures . . . . .	223
5.3.1	The Chaos Game : 'Heads' , 'Tails' and 'Side' . . . . .	223
5.3.2	How two ivy leaves lying on a sheet of paper can specify an affine transformation . . . . .	227
5.4	The computation of images from IFS codes . . . . .	228
5.4.1	What an IFS code is . . . . .	228
5.4.2	The underlying model associated with an IFS code . .	229
5.4.3	How images are defined from the underlying model . .	230
5.4.4	The algorithm for computing rendered images . . . . .	231
5.5	Determination of IFS codes: The Collage Theorem . . . . .	233
5.6	Demonstrations . . . . .	238
5.6.1	Clouds . . . . .	238
5.6.2	Landscape with chimneys and smoke . . . . .	238
5.6.3	Vegetation . . . . .	239
<b>A</b>	<b>Fractal landscapes without creases and with rivers</b>	<b>243</b>
	<i>Benoit B.Mandelbrot</i>	
A.1	Non-Gaussian and non-random variants of midpoint displacement	244
A.1.1	Midpoint displacement constructions for the paraboloids	244
A.1.2	Midpoint displacement and systematic fractals: The Tak- agi fractal curve, its kin, and the related surfaces . . . . .	246

A.1.3	Random midpoint displacements with a sharply non-Gaussian displacements' distribution . . . . .	248
A.2	Random landscapes without creases . . . . .	250
A.2.1	A classification of subdivision schemes: One may displace the midpoints of either frame wires or of tiles . . .	250
A.2.2	Context independence and the "creased" texture . . . . .	251
A.2.3	A new algorithm using triangular tile midpoint displacement	252
A.2.4	A new algorithm using hexagonal tile midpoint displacement	254
A.3	Random landscape built on prescribed river networks . . . . .	255
A.3.1	Building on a non-random map made of straight rivers and watersheds, with square drainage basins	255
A.3.2	Building on the non-random map shown on the top of Plate 73 of "The Fractal Geometry of Nature" . . . . .	258
<b>B</b>	<b>An eye for fractals</b>	<b>261</b>
	<i>Michael McGuire</i>	
<b>C</b>	<b>A unified approach to fractal curves and plants</b>	<b>273</b>
	<i>Dietmar Saupe</i>	
C.1	String rewriting systems . . . . .	273
C.2	The von Koch snowflake curve revisited . . . . .	275
C.3	Formal definitions and implementation . . . . .	279
<b>D</b>	<b>Exploring the Mandelbrot set</b>	<b>287</b>
	<i>Yuval Fisher</i>	
D.1	Bounding the distance to M . . . . .	288
D.2	Finding disks in the interior of M . . . . .	294
D.3	Connected Julia sets . . . . .	296
	<b>Bibliography</b>	<b>297</b>
	<b>Index</b>	<b>307</b>