

Contents

Preface

xxvii

CHAPTER 1

The Phenomenon of Diffraction

1. Introduction	1
2. Elementary Scalar Wave Theory (Acoustic Waves).	8
3. Elementary Vector Wave Theory	10
3.1. Equations and Units	10
3.2. The Vector and Scalar Potentials	11
3.3. Fields with Time Dependence $e^{i\omega t}$	12
4. Plane Wave Excited Fields	14
5. Axially Symmetric Fields	16
6. The Sphere Diffraction Problem	18
6.1. The Acoustic Case	18
6.2. The Electromagnetic Case	20
7. Solution of the Sphere Diffraction Problem by Means of an Infinite Legendre Integral Transform	23
7.1. The Acoustic Case	24
7.2. The Electromagnetic Case	25

CHAPTER 2

The Diffraction of Electromagnetic Waves from Regions of Small Refrangibleability

1. Introduction	26
1.1. The General Problem	26
2. Undisturbed Field	27
3. Field Inside Diffracting Region.	27
4. Field Outside the Diffracting Region	29
5. The Solution Outside the Diffracting Region	29
6. The Solution Inside the Diffraction Region	30
7. "Explicit Expression of Field Outside Diffracting Region in Terms of a Volume Integral Taken Through Diffracting Region"	30
8. The Physical Meaning of This Solution and Its Application	31

CHAPTER 3

The Jeffreys (W.K.B.) Approximation

1. Introduction	34
2. On the Order of Magnitude of the Error	40
2.1. On an Upper Bound for the Relative Error	40
2.2. The Behavior of the W.K.B. Expression in Region I	43
2.3. Examination of the W.K.B. Approximation in Region I	44
3. The Connection Formulas	49
3.1. The Determination of the Coefficients in the Connection Relations	51
3.2. Application of this Theory to the Equation $W'' + zW = 0$	53
4. Connection of the W.K.B. Approximation with the Series Approximation Valid Sufficiently Near a Zero of $f(z)$	56

CHAPTER 4

Radio Wave Propagation: Explanation of the Problem and the Historical Background

1. The General Problem	61
2. HF and VHF Waves	61
3. Meter Wave Anomalous Propagation	63
3.1. Atmospheric Inversions	63
3.2. Early Work: The Admiralty Trials	64
4. The Progress of Theory	66
4.1. The "Effective Radius" Rule	66
4.2. The "Earth-Flattening" Technique	67

CHAPTER 5

Radio Wave Propagation: The Extension of Watson's Analysis for a Nonhomogeneous Atmosphere

1. Introduction	69
2. The General Field Equations	70
2.1. Vertical Electric Dipole	71
2.2. Boundary Conditions at the Earth's Surface	73
2.3. Horizontal Electric Dipole	75
2.4. Boundary Conditions at the Earth's Surface	79
2.5. Cases in Practice when K_e Can Be Large	80

CHAPTER 6

Radio Wave Propagation: Surface Inversions

1. Introduction	82
2. The Mathematical Expression for the Field Above a Perfectly Conducting Earth	83
2.1. The Form of the Field Series (15)	86
3. The Function $u_{\nu-1/2}(\xi)$	87
3.1. Case I. $m < 1$	89

3.2. Case IA. $x^{2/3} \delta_0$ Large	90
3.3. Case IB. $x^{2/3} \delta_0$ Small	95
3.4. Summary of the Case $m < 1$	96
3.5. Case II. $m = 1$	96
3.6. Case III. $m > 1$	99
3.7. The W.K.B. Analysis and the Second Approximation to the Zeros β_1 and β_2 of $f(\rho)$	102
3.8. An Upper Bound for $\text{Im}(x\varepsilon)$	108
4. Conclusion	110

CHAPTER 7

Radio Wave Propagation: High Level Inversions

1. Introduction	112
2. The Mathematical Model.	113
3. Reflection from a High Level Inversion	114
3.1. The Inversion Profile	117
3.2. Thin Layers ($\vartheta\chi \ll 1$)	118
3.3. Thick Layers ($\vartheta\chi > 1$)	118
4. The Field Beneath an Elevated Inversion	120
4.1. The Undisturbed Field in a Homogeneous Atmosphere	120
4.2. The Field inside the Inversion	121
4.3. The Boundary Conditions	122
4.4. The Total Field below the Inversion	123
5. Approximate Evaluation of the Field	124
5.1. The Approximations to $\zeta_s(\xi)$ and $\eta_s(\xi)$	126
5.2. Connection with Ray Theory	127
6. "Thin" and "Thick" Layers	128
7. Thin Layers—Meter Wave Propagation	129
8. The Characteristic Roots.	130
8.1. The Equation for ε	130
8.2. The Roots of (70) ($X > 1$)	131
8.3. The Roots of (71) $\{X < -1\}$	133
8.4. The Roots of (72)	134
9. Series for the Field	135
10. Note on the Applicability of the Theory	137
11. Criteria for "Good" Anomalous Propagation Near, and Beyond, the Limiting Distance	139
Notes on Chapter 7.	139
1. Approximation to Normal Propagation	139
2. Ray Theory Interpretation of Characteristic Roots	140

CHAPTER 8

The Diffraction of Electric Waves Around a Finite, Perfectly Conducting Cone

1. Introduction and Mathematical Formulation of the Problem	141
2. The Equations for the Field Series Coefficients	149

3.	The Equation for the Region II Field Series Coefficients B_n . . .	154
4.	The Field Singularities	156
4.1.	The Field Near the Cone Tip	156
4.2.	The Field Near the Cone Rim	157
5.	The Edge Condition and the Determination of the Solution . . .	163
5.1.	The Determination of Π	163
5.2.	The Edge Condition	164
5.3.	The Determination of the Solution	166
6.	Conclusion	170

CHAPTER 9

Diffraction by Large and Small Cones

1.	Introduction	172
2.	The Preliminary Analysis	173
3.	The Approximation to the Solution for kl Large.	178
3.1.	The Approximation for $\Lambda(\nu_p, \nu_a)$	178
3.2.	The Solution for the $D_{\nu-1/2}$	180
3.3.	The Satisfaction of the Edge Condition	186
4.	The Determination of the Field for Large kl	189
4.1.	The Expression of Π in Terms of the C_n	189
4.2.	The Shadow Region.	189
4.3.	The Field Outside the Shadow Region	191
5.	The Solution for Small Cones ($kl \ll 1$)	203
6.	Conclusion	204

CHAPTER 10

Linear Integral Equations

1.	Introduction	206
2.	Standard Fredholm Theory	206
3.	Standard Theory Continued. The Existence of Eigenfunctions .	209
4.	The Linear Integral Equation for $\lambda = \lambda_0$ as a Limiting Case of the Standard Nonhomogeneous Linear Integral Equation When $\lambda \rightarrow \lambda_0$	210
4.1.	Explanation of the Method	210
5.	On Necessary and Sufficient Conditions for the Existence of a Limit Function $\lim_{\lambda \rightarrow \lambda_0} f(x, \lambda)$ for Theorem 1	215
5.1.	The Case $F(x, \lambda_0) \neq 0$	215
5.2.	The Case $F(x, \lambda_0) \equiv 0$	216
5.3.	Formulation of Necessary and Sufficient Conditions for Existence of $\lim_{\lambda \rightarrow \lambda_0} f(x, \lambda)$	217
6.	On Some Important Properties of the $g_i(x, y)$	223
7.	A General Integral Representation	227
8.	The λ -Polynomials	229
8.1.	The λ -Polynomial Analysis	229
8.2.	The Nature of the λ -Derivatives $F^{(i)}(x, \lambda_0)$	233
9.	The General Solution of the Nonhomogeneous Equation for $\lambda = \lambda_0$	250
10.	On the Necessary and Sufficient Condition for the Nonhomogeneous Equation to Have a Solution in the Case $\lambda = \lambda_0$	254

11. Necessary and Sufficient Condition for all Solutions of Homogeneous Equation (1) with $\lambda = \lambda_0$ To Be in the Simple Integral Form (17)	259
12. The Case of a Symmetric Kernel	260
13. On the Index of the Associated Equation	262
14. Miscellaneous Results	268
14.1. Solution of Nonhomogeneous Equation for $\lambda = \lambda_0$ when the Free Function Is a Solution of (1) with $\lambda = \lambda_0$	268
14.2. Expressions for $g_l(x, y)$ and λ_0 (When λ_0 Is the Only Eigenvalue)	269
14.3. A Relation between the Pairs $g_i(x, y); g_j(x, y)$	271

CHAPTER 11

The Simple Microwave Lens

1. The Kirchhoff-Huygens Diffraction Theory	273
1.1. Scalar Theory	273
1.2. Vector Theory	277
2. Application of the Kirchhoff Theory to the Simple Microwave Lens	278
2.1. Introduction	278
2.2. The Field in the Focal Neighborhood	280
2.3. The Energy Flow in the Neighborhood of the Focus	305
2.4. The Field Well Beyond the Focus ("Far Field")	310

CHAPTER 12

Diffraction from Objects in the Focal Field: The Slab

1. Introduction	323
2. Physical Interpretation of the Solution	327
2.1. The Field inside the Slab	327
2.2. The Field outside the Slab	330
3. Approximations to the Field	330
3.1. The Field inside the Slab ($-\tau < z < \tau$)	330
3.2. The Field outside the Slab ($z \geq \tau$)	331
4. The Far Field	334

CHAPTER 13

Diffraction from Objects in the Focal Field: The Cylinder

1. The Field in the Focal Region	337
2. The Field Well Beyond the Focus	346
2.1. Thin Cylinders ($kc \ll 1$)	347
2.2. Thick Cylinders ($kc \gg 1$)	348
2.3. Modification for a Tapered Lens	355
2.4. The Total Far Field: Comparison with Undisturbed Far Field	359

CHAPTER 14

The Fabry-Perot Microwave Interferometer: Basic Theory

1. Introduction	363
2. Basic Theory	367

3.	Expressions for the Field Inside the Cavity	377
3.1.	The Field in the Central Regions	378
4.	The Force-Free (Resonance) State	379
5.	Conclusion	381

CHAPTER 15

The Concentric Resonator

1.	Introduction	383
2.	The First Approximation to the Solution Function $G(T)$	386
2.1.	Note on the Case when μ May Be Large	388
3.	The Resonant Frequencies	389
4.	The Second Approximation	390
4.1.	The Case when m and n Are Large	395
4.2.	The Determination of the Eigenvalues	396
5.	Estimation of the Error in the Second Approximation: the "Ripple" Terms	398
6.	The Attenuation Per Plate Reflection	404
7.	Comparison with Existing Theory and with Experiment	405
8.	Extension of the Theory to Nonaxially Symmetric Excited Modes	406

CHAPTER 16

The Ex-Centric Resonator

1.	Introduction	407
2.	The Exact Solution of Integral Equation (6)	410
3.	The Approximation to $F(T)$ for Large C_1	412
3.1.	The Expression of $F(T)$ as a Series of Elementary Eigenfunctions	412
3.2.	The Determination of the $e_{nr}(T)$ ($r = 1, 2, \dots$)	413
4.	The Second Approximation to the Eigenvalue K_n	422
4.1.	The Relation between K_n and $G_n(1)$	422
4.2.	The Attenuation per Plate Reflection	426
5.	Extension of This Solution for Nonaxially Symmetric Modes	426
5.1.	The Second Approximation to K_{nN}	432
6.	The Solution When δ/f Is Very Small	433
6.1.	The Intermediate Case $C^{-1} \ll \gamma \ll 1$	437
6.2.	Extension of the Above for Nonaxially Symmetric Modes	439

CHAPTER 17

The Driven Microwave Interferometer

1.	Recapitulation	442
2.	The Determination of the Plate Distribution Function	442
2.1.	Case $C \ll 1$ (Narrow Beam Width Case)	443
2.2.	Case $C \gg 1$ (Wide Beam Width Case)	444
3.	The Transmission Coefficient for the Interferometer in the "Concentric" Case	448
4.	The Field in the Central Regions of the Interferometer	449
4.1.	Use of the Theory of Chapter 14, Section 3.2	449
4.2.	The Concentric Case	450
4.3.	The Ex-centric Case	453

APPENDIX

Chapter 1: The Phenomenon of Diffraction

1-1. Form of Ω in Section 4 for a Plane Wave in Direction z_0	457
1-2. Expansion of Ω for Plane Wave in Spherical Wave Functions	458
1-3. An Infinite Legendre Integral Transform and Its Inverse	459
1-4. The Expansion Theorem (72) for $(r^2 + f^2 - 2rf \cos \theta)^{-1/2} \exp\{-ik \cdot (r^2 + f^2 - 2rf \cos \theta)^{1/2}\}$	465

Chapter 3: The Jeffreys (W.K.B.) Approximation

3-1. On the Requirements for the One-Valuedness of the Inverse Function to $F(z)$	470
3-2. On the Use of Approximation (35) in (33)	472

Chapter 4: Radio Wave Propagation: Explanation of the Problem and the Historical Background

4-1. Note on the So-called "Phase Integral" Method	473
----------------------------------------------------	-----

Chapter 5: Radio Wave Propagation: The Extension of Watson's Analysis for a Nonhomogeneous Atmosphere

5-1. On the Value of Π for the Undisturbed Dipole Field when K Varies	474
-----------------------------------------------------------------------------	-----

Chapter 6: Radio Wave Propagation: Surface Inversions

6-1. Proof that $\text{Im}(\epsilon)$ Is Always Negative	481
6-2. The Error in Approximating an Exponential Function by the First Few Terms of its Series	482
6-3. Checks on the Satisfaction of the W.K.B. Sufficiency Condition	482
6-4. On the Trapping of Rays Leaving the Earth's Surface	485
6-5. The Continuation Relations for the W.K.B. Approximation $f^{-1/4}\{Ae^{iF} + Be^{-iF}\}$ where the Plane of the Independent Variable Is Cut To Make $-\pi < \arg F < 2\pi$	486
6-6. The Lower Bound of $ q $ on the Imaginary Axis	487

Chapter 7: Radio Wave Propagation: High Level Inversions

7-1. The Factor $\eta_{\nu-1/2}(x_1)(\partial/\partial s)\{\zeta_{s-1/2}(x_1) - k\eta_{s-1/2}(x_1)\}_{(s=\nu)}$	488
7-2. The Factor \mathcal{F}_j	489

Chapter 8: The Diffraction of Electric Waves Around a Finite, Perfectly Conducting Cone

8-1. The Solution for a Semi-infinite Cone	491
8-2. The Fixing of the Wave Function Π	492
8-3. Value of K in (3)	494
8-4. Uniqueness of the Equations for the A_j and B_n	494
8-5. Two Legendre Function Theory Results	496
8-6. The Behavior of $C_{s-1/2}$ for Large s	496
8-7. On the Electric Vector near the Cone Rim	504

Chapter 9: Diffraction by Large and Small Cones

9-1. The Approximation to $\sum_{j=0}^{\infty} \frac{(2\beta_j + 1)\psi_{\beta_j}\zeta'_{\beta_j}}{\beta_j(\beta_j + 1)\{v^2 - (\beta_j + \frac{1}{2})^2\}} \frac{P'_{\beta_j}(-\mu_0)}{[\partial P_s(-\mu_0)/\partial s]_{(\beta_j)}}$	504
-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----

9-2. The Sum $\sum \frac{2\beta_j + 1}{\{\nu^2 - (\beta_j + \frac{1}{2})^2\}[\partial P_s(-\mu_0)/\partial s]_{(\beta_j)}} \zeta_{\beta_j} \psi_{\beta_j}$	507
9-3. The Approximation to $A(\nu_p, \nu_q)$	509
9-4. Solution of Integral Equation (39) of the Text	512
9-5. Proofs of the Integrals (52) in Chapter 9	518
9-6. The Sums in (95) of Chapter 9	520
9-7. The Approximation to Π on the Cone Base	531
Chapter 13: Diffraction from Objects in the Focal Field: The Cylinder	
13-1. Approximations to A_n and B_n when $kc \ll 1$	532
13-2. A Certain Infinite Integral	534
13-3. Focused Microwaves Incident Obliquely on a Uniform Dielectric Cylinder	535
13-4. Imaging of a Rectangular Source by a Focused Lens and Scattering by a Dielectric Cylinder Placed across the Focal Region Thereof	549
13-5. Evaluation of the Scattered Power	556
13-6. The Limit of $\rho \iint E_{x_0} \Delta E_x dZ d\phi$ as $\rho \rightarrow 0$	557
Chapter 14: The Fabry-Perot Microwave Interferometer: Basic Theory	
14-1. The Reflection Hypothesis	559
Chapter 15: The Concentric Resonator	
15-1. The Relation of $J_0(\mu T)$ to $G(T)$ for large μ	560
15-2. The Expression of the Double Integral in the Equation for b_n in Terms of Another Whose Range of Integration Excludes the Origin	562
15-3. The Approximation to $\phi(j_m, j_n)$ and $\phi(j_n, 0)$ when C Is Large but j_m, j_n Are Not Large	565
15-4. The Calculation of $\phi(k, 0)$ for Large C When k Is Not Large	577
15-5. Calculation of $\phi(j_m, j_n)$ and $\phi(j_n, 0)$ When C Is Large and Also j_m and j_n Are Large.	582
Chapter 16: The Ex-Centric Resonator	
16-1. Generalization of the Function $F_p(u, v)$ of Chapter 11	589
16-2. Calculation of $\square_{hk} \phi _{(h=j_m, k=j_n)}$	593
16-3. Calculation of $((\partial^2/\partial k^2) + (1/k)(\partial/\partial k))\phi(k, 0) _{(k=j_n)}$	597
Chapter 17: The Driven Microwave Interferometer	
17-1. Transformation of Sums for ϕ_1 and ϕ_2 of Chapter 17	597
17-2. Proof of the Lemma (60)	599
Mathematical Supplement	600
References	624
Author Index	627
Subject Index	629

