

CONTENTS

<i>Editor's Preface</i>	ix
<i>Foreword</i>	xi
<i>Introduction: Historical Review</i>	xv

I. THE OPERATORS OF MATHEMATICAL PHYSICS

§ 1. The formulation of the basic problems	1
§ 2. Some ancillary concepts and formulae	6
§ 3. The scalar product of functions	12
§ 4. The concepts of operator and of functional	20
§ 5. Symmetric operators	25
§ 6. Positive definite and positive-bounded-below operators	31

II. ENERGY CONVERGENCE

§ 7. The estimation of the error of an approximation and the nature of its convergence. Convergence in the mean	40
§ 8. Energy convergence	51
§ 9. The linear independence of functions	56
§ 10. Orthogonality and orthogonal series	59

III. THE ENERGY METHOD

§ 11. The minimum functional theorem	74
§ 12. Representation of a solution in the form of an orthogonal series	80
§ 13. The minimizing sequence and its convergence	83
§ 14. The Ritz method	85
§ 15. Other methods of constructing the minimizing sequence	95
§ 16. Functions with finite energy	100
§ 17. The applications of functions with finite energy. Natural boundary conditions	109
§ 18. Non-homogeneous boundary conditions	116
§ 19. The existence of a solution of the variational problem	121

IV. MAJOR APPLICATIONS OF THE ENERGY METHOD

§ 20. Boundary value problems for an ordinary differential equation	126
§ 21. Bending of a beam of variable cross-section, resting on an elastic foundation	136

§ 22. Fundamental boundary value problems for the equations of Poisson and Laplace	138
§ 23. Problems of torsion of a bar and of bending of a beam by a lateral force	151
§ 24. Equations with variable coefficients	160
§ 25. Degenerate elliptic equations; the Chaplygin equation	167
§ 26. The principle of minimum of potential energy in elasticity theory	175
§ 27. The bending of thin plates	182
§ 28. The bending of thin plates to which both normal loads and loads acting in the central plane are applied	200

V. THE EIGENVALUE PROBLEM

§ 29. The eigenvalue problem; its connexion with characteristic vibrations and stability of a system	207
§ 30. Eigenvalues and eigenfunctions of a symmetric operator	213
§ 31. Energy theorems in the eigenvalue problem	218
§ 32. Ritz's method for the eigenvalue problem	226
§ 33. Another form of Ritz's method; the case of natural boundary conditions	233
§ 34. Equations of the form $Au - \lambda Bu = 0$	236
§ 35. The eigenvalues of an ordinary differential equation	239
§ 36. The stability of a compressed rod	250
§ 37. Eigenvalues of elliptic operators	253
§ 38. The stability of a compressed plate	262
§ 39. The characteristic vibrations of an elastic body	265
§ 40. The minimax principle and its implications	270

VI. GENERALIZATION OF THE PRECEDING RESULTS

§ 41. The concept of Lebesgue integral	278
§ 42. Functional Hilbert spaces	287
§ 43. The limit process in Hilbert spaces	295
§ 44. Generalization of the concept of orthogonality	300
§ 45. General definitions of functional and operator	308
§ 46. The general formulation of the variational problem and its solution	318
§ 47. The method of minimum surface integrals	328

VII. ESTIMATES OF THE ERROR OF AN APPROXIMATE SOLUTION

§ 48. General remarks	332
§ 49. Subspaces and projections	335
§ 50. The method of orthogonal projections in Dirichlet's problem	339
§ 51. General formulation of the method of orthogonal projections	345

§ 52. Some additional considerations	350
§ 53. The Neumann problem	352
§ 54. Castigliano's principle and two-sided estimates in elasticity theory	355
§ 55. Trefftz's method	359
§ 56. The biharmonic equation. The method of anharmonic residue	365
§ 57. Generalization of Trefftz's method	369
§ 58. Application to Poisson's equation	371
§ 59. Generalization of Trefftz's method to the problem of a freely supported plate	375
§ 60. The method of M. G. Slobodyansky	380
§ 61. Upper and lower bounds for functionals	383
§ 62. Upper and lower bounds of eigenvalues	385
§ 63. Estimate of the error due to a perturbation of the equation	391

VIII. NUMERICAL EXAMPLES

§ 64. On coordinate functions	395
§ 65. The torsion of a rod of rectangular cross-section	402
§ 66. Bending of a rectangular plate, rigidly fixed at the rim	413
§ 67. Bending of semicircular plate elastically supported at the rim	419
§ 68. Evaluation of the eigenvalues of an ordinary second-order differential equation	423
§ 69. Eigenvibrations of a beam of variable cross-section	427
§ 70. Radial eigenvibrations of an elastic cylinder	435
§ 71. Plane vibrations of an elastic rectangular plate	440
§ 72. Stability of a compressed elliptic plate	445

IX. THE BUBNOV—GALERKIN METHOD

§ 73. Foundations of the method	448
§ 74. Proof of convergence for Fredholm integral equations	450
§ 75. Proof of convergence for ordinary second-order differential equations	455
§ 76. Completely continuous operators	459
§ 77. Equations involving a completely continuous operator	462
§ 78. A sufficient criterion for the convergence of the Bubnov—Galerkin method	469
§ 79. Application to ordinary differential equations	477
§ 80. The Dirichlet problem for a second-order elliptic equation	480
§ 81. The Neumann problem and the mixed problem for a second-order elliptic equation	485
§ 82. Modification of the Bubnov—Galerkin method for the case of natural boundary conditions	489

X. THE METHOD OF LEAST SQUARES

§ 83. Basis of the method .	491
§ 84. Application to integral equations	499
§ 85. Application to boundary value problems with homogeneous boundary conditions	503
§ 86. Auxiliary theorems of the theory of analytic functions	504
§ 87. The Dirichlet and Neumann problems	509
§ 88. The Dirichlet problem for an ellipse	514
§ 89. The case of a piecewise smooth contour. The Dirichlet problem	515
§ 90. The mixed problem of potential theory	518
§ 91. The plane problem of elasticity theory	526
§ 92. The periodic problem of elasticity theory	530
§ 93. Stress in an elastic space bounded by a sine curve	538

XI. FINITE DIFFERENCE METHODS

§ 94. The method of networks	544
§ 95. Foundations of the method of straight lines	549
§ 96. Differential equations of the method of straight lines for the equations of Poisson and Laplace	551
§ 97. The case of a trapezoidal domain	554
§ 98. Differential equations of the method of straight lines for the biharmonic equation	563

Bibliography 567

Index 577

List of Volumes in the Series in Pure and Applied Mathematics 583

