

Table of Contents

Preface	V
Color Plates	XVII
1 Bifurcations Observed from Electronic Circuits	1
1.1 Introduction	1
1.2 The Double Scroll Circuit	2
1.2.1 Circuit and its Dynamics	2
1.2.2 Implementation	4
1.2.3 Experiments	7
A Hopf Bifurcation	7
B Period-Doubling Bifurcations of the Periodic Orbit	7
C Chaotic Attractor (Rössler's Spiral-type)	7
D Saddle-Node Bifurcations of the Periodic Orbit and Periodic Window	8
E Interior Crisis (The Double Scroll)	8
F Near Heteroclinicity	8
G Boundary Crisis	8
H Sounds	9
1.2.4 Confirmations	10
A Hopf Bifurcation	10
B Period-Doubling Bifurcation	11
C Chaotic Attractor (Rössler's Spiral-type)	11
D Saddle-Node Bifurcation and Periodic Window . .	15
E Interior Crisis (The Double Scroll)	16
F Near Heteroclinicity	16
G Boundary Crisis	16
1.2.5 Summary	17
1.3 Structure of the Double Scroll	20
1.3.1 Geometric Structure	20
1.3.2 Lyapunov Exponents and Lyapunov Dimension	29
A Lyapunov Exponents	29

	B	Computations	32
	C	Explicit Formula	34
	D	Lyapunov Dimension	35
	E	Time Waveforms and Power Spectra	35
1.4		The Double Scroll Circuit is Chaotic in the Sense of Shil'nikov . .	35
1.4.1		Statement	35
1.4.2		The Class \mathcal{L}	37
1.4.3		Equivalence and Conjugacy Classes of \mathcal{L}	44
1.4.4		Subset \mathcal{L}_{DS}	50
	A	Half-Return Map π_0	51
	B	Half-Return Map π_1	56
	C	The Map Φ	59
	D	Poincaré Map π	60
1.4.5		Completion of the Proof	62
1.5		Homoclinic Linkage	74
1.5.1		Introduction	74
1.5.2		Bifurcation Equations	74
	A	Normal Form	74
	B	Return Time Coordinates	76
	C	Periodic Orbits	78
	D	Bifurcation Conditions for Periodic Orbits	78
	E	Homoclinic Orbits Passing Through O	80
	F	Homoclinic Orbits Passing Through P^+	80
	G	Heteroclinic Orbits	81
1.5.3		Global Bifurcations	82
	A	Homoclinic/Heteroclinic Bifurcation Sets	83
	B	Homoclinic Linkage	90
	C	Global Bifurcations of Periodic Windows	96
1.6		The Torus Breakdown Circuit	102
1.6.1		Introduction	102
1.6.2		Observations of Torus Breakdown	102
	A	The Circuit and its Dynamics	102
	B	Experiments	104
	C	Period-Adding Sequence	109
	D	Sounds	110
1.6.3		Analysis	110
	A	Divergence Zero Boundary	111
	B	Trajectories on the Torus	112
	C	The Folded Torus and the Double Scroll	114
1.7		The Hyperchaotic Circuit	115
1.7.1		Introduction	115
1.7.2		Experiment	115
	A	Observation	115
	B	Sounds	116
1.7.3		Confirmation	116

1.8	The Neon Bulb Circuit	120
1.8.1	Introduction	120
1.8.2	Experiment	121
	A Observation	121
	B Sounds	122
1.8.3	Arnold Tongues	122
1.8.4	Rotation Numbers	123
1.9	The <i>R-L</i> -Diode Circuit	124
1.9.1	Experiment 1	124
1.9.2	Analysis 1	125
	A The Dynamics	125
	B Two-Dimensional Map Model	129
	C The Bifurcation Scenario	129
1.9.3	Experiment 2	133
1.9.4	Analysis 2	133
2	Bifurcations of Continuous Piecewise-Linear Vector Fields	139
2.1	Introduction	139
2.2	Definition and Standard Forms of Continuous Piecewise-Linear Maps	140
2.2.1	Definition of Piecewise-Linear Maps	140
2.2.2	Standard Forms of CPL Maps with the Boundary Set in General Position	142
2.2.3	Standard Forms of CPL Functions	146
2.2.4	Examples of CPL functions	157
2.3	Normal Forms of Piecewise-Linear Vector Fields	163
2.3.1	Notations	165
2.3.2	Normal Forms of Linear Vector Fields with a Boundary	180
2.3.3	Normal Forms of Degenerate Affine Vector Fields with a Boundary	191
2.3.4	Normal Forms of Two-Region Piecewise-Linear Vector Fields	210
2.3.5	Normal Forms of Proper Two-Region Piecewise-Linear Vec- tor Fields	237
2.4	Multiregion Systems and Chaotic Attractors	244
2.4.1	Attractors in Three-Dimensional Three-Region System	244
2.4.2	The Piecewise-Linear Lorenz Attractor	249
2.4.3	The Piecewise-Linear Duffing Attractor	254
2.5	Bifurcation Equations of Piecewise-Linear Vector Fields	257
2.5.1	Normal Forms of Three-Dimensional Two-Region Systems	258
2.5.2	The Tangent Map of Poincaré Full Return Maps	264
2.5.3	The Return Time Coordinates	267
2.5.4	Bifurcation Equations of Three-Dimensional Two-Region Systems	269
	A Homoclinic Bifurcations	270
	B Heteroclinic Bifurcations	272
2.5.5	Bifurcation Equations of Periodic Orbits	274

2.6	Bifurcation Sets	278
2.6.1	Homoclinic/Heteroclinic Bifurcation Sets	279
	A Bifurcation Sets for Principal Homoclinic Orbits .	279
	B Subsidiary Homoclinic Bifurcation Sets and Hete- roclinic Bifurcation Sets	281
2.6.2	Bifurcation Sets for Periodic Orbits	281
	A Saddle-Node Bifurcation Sets	286
	B Period-Doubling Bifurcation Sets	286
	C Windows	286
2.6.3	Computing Bifurcation Sets	294
3	Fundamental Concepts in Bifurcations	297
3.1	Introduction	297
3.2	Fundamental Notions for Dynamical Systems	299
3.2.1	Definitions and Examples of Dynamical Systems	299
3.2.2	Orbits and Invariant Sets in Dynamical Systems	305
3.2.3	Linearization at Equilibrium Points and the Theorem of Hartman-Grobman	312
3.2.4	Stable and Unstable Manifolds	318
3.2.5	Topological Equivalence and Structural Stability	323
3.2.6	Bifurcation	326
3.2.7	Framework for the Bifurcation Theory	329
3.3	Local Bifurcations around Equilibrium Points in Vector Fields . . .	330
3.3.1	Center Manifolds	330
3.3.2	Normal Forms	338
3.3.3	Codimension One Bifurcations	343
	A Saddle-Node Bifurcation	343
	B Hopf Bifurcation	346
3.3.4	Bogdanov-Takens Bifurcation	351
3.3.5	Symmetry and Bifurcations	355
3.3.6	Other Degenerate Singularities	360
3.4	Dynamics and Bifurcations for Discrete Dynamical Systems	362
3.4.1	Discrete Dynamical Systems	362
3.4.2	Basic Theorems and Structural Stability	367
3.4.3	Elementary Bifurcations	368
	A Saddle-Node Bifurcation	368
	B Period-Doubling Bifurcation	369
	C Hopf Bifurcation	371
3.4.4	One-Dimensional Mapping (1)	373
	A Elementary Bifurcations for Quadratic Family . .	374
	B The Case of $\mu < -2$	376
	C The Case of $\mu = -2$	378
3.4.5	One-Dimensional Mapping (2)	380
3.4.6	Horseshoe	384
	A Topological Horseshoe	384

	B	Hyperbolicity	386
	C	Transverse Homoclinic Points and Horseshoes . . .	390
3.4.7		Further Developments	393
	A	One-Dimensional Quadratic Family	393
	B	Lozi Map	395
	C	Hénon Map	395
	D	Homoclinic Tangency	396
3.5		Bifurcations of Homoclinic and Heteroclinic Orbits in Vector Fields	398
3.5.1		Persistence of Homoclinic/Heteroclinic Orbits and the Melnikov Integral	398
3.5.2		Shil'nikov Theorem	406
3.5.3		Gluing Bifurcations for Heteroclinic Orbits and Exponential Expansion	411
3.5.4		T-points and Gluing Bifurcations with Different Saddle-Indices	417
3.5.5		Homoclinic Doubling Bifurcation	421
	A	Motivation	421
	B	Homoclinic Doubling Bifurcation Theorems	424
	C	Proof of the Homoclinic Doubling Bifurcation Theorems	427
	D	Further Development	431
3.5.6		Bifurcation Generating Geometric Lorenz Attractors from Homoclinic Orbits	432
3.5.7		Local Bifurcations and Global Bifurcations	441
		References	445
		Index	459
		Credits	467