CONTENTS

Foreword V

Chapter 1
Introduction and Notation 1
1.1 Introduction 1
1.2 Notation 7

Chapter 2
Ergodicity 11
2.1 Anosov's result 11
2.2 Method of proof 14
2.3 Proof of Lemma 1 15
2.4 Proof of Lemma 2 23

Chapter 3
On Frequency Systems and First Result for Two Frequency Systems .. 25
3.1 One frequency; introduction and first order estimates 25
3.2 Increasing the precision; higher order results 35
3.3 Extending the time-scale; geometry enters 42
3.4 Resonance; a first encounter 54
3.5 Two frequency systems; Arnold's result 56
3.6 Preliminary lemmas 58
3.7 Proof of Arnold's theorem 61

Chapter 4
Two Frequency Systems; Neistadt's Results 67
4.1 Outline of the problem and results 67
4.2 Decomposition of the domain and resonant normal forms 71
4.3 Passage through resonance: the pendulum model 78
4.4 Excluded initial conditions, maximal separation, average separation 92
4.5 Optimality of the results 101
4.6 The case of a one-dimensional base 111

Chapter 5
N Frequency Systems; Neistadt's Result Based on Anosov's Method ... 117
5.1 Introduction and results 117
5.2 Proof of the theorem 120
5.3 Proof for the differentiable case 126

Chapter 6
N Frequency Systems; Neistadt's Results Based on Kasuga's Method 133
6.1 Statement of the theorems 133
6.2 Proof of Theorem 1 135
6.3 Optimality of the results of Theorem 1 142
6.4 Optimality of the results of Theorem 2 148

Chapter 7
Hamiltonian Systems 153
7.1 General introduction 153
7.2 The KAM theorem 154
7.3 Nekhoroshev's theorem; introduction and statement of the theorem 161
7.4 Analytic part of the proof 164
7.5 Geometric part and end of the proof 173

Chapter 8
Adiabatic Theorems in One Dimension 183
8.1 Adiabatic invariance; definition and examples 183
8.2 Adiabatic series 199
8.3 The harmonic oscillator; adiabatic invariance and parametric resonance 205
8.4 The harmonic oscillator; drift of the action 212
8.5 Drift of the action for general systems 216
8.6 Perpetual stability of nonlinear periodic systems 222

Chapter 9
The Classical Adiabatic Theorems in Many Dimensions 229
9.1 Invariance of action, invariance of volume 229
9.2 An adiabatic theorem for integrable systems 230
9.3 The behavior of the angle variables 237
9.4 The ergodic adiabatic theorem 239

Chapter 10
The Quantum Adiabatic Theorem 249
10.1 Statement and proof of the theorem 249
10.2 The analogy between classical and quantum theorems 252
10.3 Adiabatic behavior of the quantum phase 257
10.4 Classical angles and quantum phase 262
10.5 Non-communtativity of adiabatic and semiclassical limits 265

Appendix 1
Fourier Series 269

Appendix 2
Ergodicity 277

Appendix 3
Resonance 281

Appendix 4
Diophantine Approximations 289

Appendix 5
Normal Forms 293

Appendix 6
Generating Functions 299

Appendix 7
Lie Series 305

Appendix 8
Hamiltonian Normal Forms 317

Appendix 9
Steepness 329

Bibliography 339

Bibliographical Notes 351

Index 357