TABLE OF CONTENTS

1. INTRODUCTION

   1-1. Outline of Course
   1-2. Hilbert Transforms
   1-3. Remarks on Analyticity in the Physics of Elementary Particles

2. PARAMETRIC REPRESENTATION OF FEYNMAN INTEGRALS

   2-1. Self-Energy Part
   2-2. The General Feynman Integral

3. CONDITIONS FOR SINGULARITIES OF INTEGRAL TRANSFORMS

   3-1. Singularities of Single Variable Integral Transforms
   3-2. Multiple Integral Transforms

4. SINGULARITIES OF FEYNMAN AMPLITUDES

   4-1. Discriminant Conditions for Singularities
   4-2. The Landau-Bjorken Conditions for Singularities
   4-3. Example, Normal Threshold of Fourth Order Amplitude

5. THE THIRD ORDER VERTEX FUNCTION

   5-1. The Reality Region
   5-2. Single Variable Dispersion Relation
   5-3. The Landau Method (and Dual Diagrams)
   5-4. Transitions from Physical to Unphysical Singularities

6. PHYSICAL INTERPRETATION OF SINGULARITIES OF THE VERTEX PART 37

6-1. Electron-Proton Scattering 37
6-2. Electron-Deuteron Scattering 38
6-3. Electron-Σ Scattering 39
6-4. Relative Momenta and Anomalous Thresholds 40

7. THE SCATTERING AMPLITUDE IN FOURTH ORDER FOR EQUAL MASSES I 42

7-1. Kinematics 42
7-2. Single Variable Dispersion Relation 44
7-3. Double Dispersion Relation by Direct Integration 45
7-4. The Mandelstam Representation 47

8. SCATTERING AMPLITUDE IN FOURTH ORDER II 47

8-1. The Parametric Integral 47
8-2. Reality Region for Equal Masses 49
8-3. Derivation of the Single Variable Dispersion Relation 49
8-4. The Landau Curves 50
8-5. The Landau Surface 53
8-6. The Double Dispersion Relation 54
8-7. Singularities on Unphysical Sheets 54

9. MANDELSTAM REPRESENTATION IN FOURTH ORDER USING ANALYTIC COMPLETION 55

9-1. Analytic Completion 55
9-2. The Disc Theorem 56
9-3. Application to the Scattering Amplitude in Fourth Order 56

10. ANOMALOUS THRESHOLDS IN FOURTH ORDER (introductory remarks only) 60

11. SINGLE VARIABLE DISPERSION RELATIONS FOR A GENERAL TERM 62

11-1. The Symanzik Region 62
11-2. The Analytic Function $A(z_1, t)$ 66
11-3. The Feynman Amplitude as a Boundary Value 67
11-4. Dispersion Relation for $A(z_1, t)$ 68
11-5. Extension of the Symanzik Region 69

12. PARTIAL WAVE DISPERSION RELATIONS FOR EQUAL MASSES 73

12-1. From the Mandelstam Representation 73
12-2. From Perturbation Theory to Any Order 77

13. UNITARITY AND EQUATIONS FOR THE PARTIAL WAVE AMPLITUDE 79

13-1. The N over D Method 79
13-2. Interpretation of the Discontinuity on the Left Hand Cut 83
13-3. Coupled Equations for Partial Waves Using Crossing Symmetry 85

14. PARTIAL WAVE DISPERSION RELATIONS FOR PION-NUCLEON SCATTERING 88

14-1. Kinematics 88
14-2. The Singularities of the Partial Wave Amplitude 92

15. CONCLUDING REMARKS 94

REFERENCES 98