

Contents

Series Foreword xi

Introduction xiii

1 Review of Stochastic Processes 1

- 1 Martingales 3
- 2 The Wiener Process and Stochastic Integrals 5
- 3 Itô's Lemma 9
- 4 Stochastic Differential Equations 11
- 5 The Martingale Problem 15
- 6 Jump Diffusion Processes 20

2 Weak Convergence 23

- 1 Introduction 23
- 2 Definitions and Motivation 25
- 3 General Weak Convergence Theorems 28
- 4 The Spaces $C^r[0, T]$ and $D^r[0, T]$, $T \leq \infty$ 30

3 Weak Convergence Methods 34

- 1 Introduction 34
- 2 The Perturbed Test Function Method 36
- 3 Tightness 47
- 4 The Discrete Parameter Case 51
- 5 Convergence of Finite-Dimensional Distributions and the Kato-Trotter Theorem 52

4 The Perturbed Test Function Method 56

- 1 Introduction 56
- 2 ODE Limits: State-Independent Noise 57
- 3 The Discrete Parameter Case: State-Independent Noise and ODE Limits 67
- 4 State-Dependent Noise and Nonsmooth Dynamics: Discrete Parameter 69
- 5 The Continuous Parameter Case: State-Dependent Noise 74
- 6 Diffusion Limits: Smooth Dynamics and Continuous Parameter 78

7	Diffusion Limits: Discrete Parameter	87
8	A Special case: A User's Guide	91
9	The Discrete Parameter Case: State-Dependent Noise, Diffusion Limits	93

5 Weak Convergence by a Direct-Averaging Method 97

1	Introduction	97
2	Noise Independent of the State: ODE Limit	99
3	Discrete Parameter Case: ODE Limit and State-Dependent Noise	105
4	A Projection Algorithm	114
5	Continuous Parameter Case: ODE Limit and State-Dependent Noise	116
6	Diffusion Limit	121
7	Jump Process Limit	123
8	The Direct-Averaging Method with Perturbed Test Functions	126
9	A Stochastic Approximation Example	138

6 Stability and Asymptotic and Invariant Measures 141

1	Introduction	141
2	Tightness and Stability for Itô Processes	144
3	Recurrence, Path Excursion Estimates, and Moment Boundedness for $x^\epsilon(\cdot)$	148
4	Asymptotic Measures of the $\{x^\epsilon(t)\}$	153
5	Convergence When $(x^\epsilon(\cdot), \xi^\epsilon(\cdot))$ Is Markov	156
6	A Sufficient Condition for Tightness; Constructing a Perturbed Liapunov Function	158
7	Nonsmooth Dynamics and/or State-Dependent Noise	163
8	Unbounded Noise	166
9	The Discrete Parameter Case	171
10	Stability w.p. 1, and Weak Convergence for Stochastic Approximations	173

7 Singular Perturbations 175

1	Introduction	175
---	--------------	-----

2	The Direct-Averaging Method	175
3	On the Stability and Tightness Assumptions	181
4	The Direct-Averaging Method with Perturbed Test Functions	187
8	Applications I	189
1	Adaptive Antenna Arrays: Weak Convergence of the Array Weights	189
2	Adaptive Antenna Arrays: Asymptotic Properties for Large Time and Normalized Errors	193
3	An Adaptive Equalizer	199
4	An Adaptive Quantizer	204
5	Rate of Convergence for Stochastic Approximations	206
9	Applications II	211
1	The Wong-Zakai Problem	211
2	A Runge-Kutta Method for Solving a Stochastic Differential Equation	214
3	The Output Process of a Hard Limiter	215
10	Applications III: Synchronization Systems	223
1	A Standard Phase-Locked Loop	224
2	A Phase-Locked Loop with a Limiter	231
11	The Theory of Large Deviations and Applications	239
1	Introduction	239
2	Basic Results: The Systems and Action Functionals	241
3	The Idea of Large Deviations and Chernoff Bounds	245
4	The Freidlin-Gartner Approach to Large Deviations Estimates	249
5	Escape Time Estimates	254
6	Applications and Examples	256
References	261	
Index	267	