



## CONTENTS

|  |    |
|--|----|
| Foreword   | v  |
| Chapter 1: INTRODUCTION  |    |
| 1.1 Dawn of Nonlinear Nonequilibrium Physics   | 2  |
| 1.2 Dawn of Chaos Physics  | 4  |
| 1.3 Onset of Chaos   | 11 |
| 1.4 Transition from Torus to Chaos<br>Accompanied by Lockings —<br>Outline of the book     | 15 |
| Chapter 2: INSTABILITY OF PHASE MOTION OF TORI   |    |
| 2.1 Introduction   | 24 |
| 2.2 Structure of Lockings  | 25 |
| 2.3 Similarity of the Period-Adding Sequences<br>of Lockings (Numerical Results)           | 27 |
| 2.4 Phenomenological Theory of the Similarity<br>of the Period-Adding Sequence             | 32 |
| 2.5 Classification of Period-Adding Sequences  | 37 |
| 2.6 Period-Adding Sequence as Windows  | 41 |
| 2.7 Scaling Properties at the Collapse of<br>Tori — A brief Review on a Recent<br>Progress | 44 |
| 2.8 Global Properties of the Devil's Staircase   | 48 |

|  |  |     |
|--|--|-----|
| 2.9  | Supercritical Behavior of Disordered Orbits of a Circle Map    | 54  |
| 2.10   | Discussion   | 68  |
|  | Appendix   | 81  |
| Chapter 3: TRANSITION FROM TORUS TO CHAOS ACCOMPANIED BY FREQUENCY LOCKINGS WITH SYMMETRY BREAKING |  |     |
| 3.1  | Introduction   | 86  |
| 3.2  | Phase Diagram and General Aspects of the Coupled-Logistic Map  | 87  |
| 3.3  | Scaling of the Period-Adding Sequence at the Frequency Locking | 93  |
| 3.4  | Frequency Locking with Symmetry Breaking                       | 96  |
| 3.5  | Discussion   | 101 |
| Chapter 4: OSCILLATION AND FRACTALIZATION OF TORI  |  |     |
| 4.1  | Introduction   | 106 |
| 4.2  | Oscillation of Torus in Two-Dimensional Mappings               | 107 |
| 4.3  | Fractalization of Torus  | 118 |
| 4.4  | Summary and Discussion   | 125 |
|  | Appendix   | 132 |
| Chapter 5: DOUBLING OF TORUS   |  |     |
| 5.1  | Discovery  | 134 |
| 5.2  | Doubling Stops by a Finite Number of Times                     | 135 |
| 5.3  | Mechanism of the Interruption of the Doubling Cascade          | 145 |
| 5.4  | Discussion   | 148 |
| Chapter 6: FATES OF THREE-TORUS  |  |     |
| 6.1  | Introduction   | 154 |
| 6.2  | Three-Torus in a Four-Dimensional Mapping                      | 155 |



|     |   |     |
|-----|---|-----|
| 6.3 | Double Devil's Staircase in<br>Modulated Circle Map | 163 |
| 6.4 | Chaos from $T^3$ in a Coupled Circle Map            | 167 |
| 6.5 | Summary and Discussions                             | 178 |

## Chapter 7: TURBULENCE IN COUPLED MAP LATTICES

|     |  |     |
|-----|--|-----|
| 7.1 | Introduction   | 186 |
| 7.2 | Period-Doublings of Kink-Antikink<br>Patterns            | 187 |
| 7.3 | Zigzag Instability and Transition<br>from Torus to Chaos | 190 |
| 7.4 | Spatiotemporal Intermittency                             | 193 |
| 7.5 | Period-Doubling in Open Flow                             | 214 |
| 7.6 | Cellular Automata  | 222 |
| 7.7 | Discussions  | 228 |

## Chapter 8: SUMMARY, FUTURE PROBLEMS, AND DISCUSSIONS

|     |  |     |
|-----|--|-----|
| 8.1 | Summary and Future Problems                                      | 236 |
| 8.2 | What has Chaos Brought about and<br>will Bring about in Science? | 242 |
| 8.3 | Towards a Field Theory of Chaos                                  | 256 |