



# TABLE OF CONTENTS

PREFACE .....	vii
TABLE OF CONTENTS .....	xi

## I STOCHASTIC VARIABLES

1 Definition .....	1
2 Averages .....	5
3 Multivariate distributions .....	10
4 Addition of stochastic variables .....	15
5 Transformation of variables .....	19
6 The Gaussian distribution .....	24
7 The central limit theorem .....	27

## II RANDOM EVENTS

1 Definition .....	32
2 The Poisson distribution .....	35
3 Alternative description of random events .....	37
4 The inverse formula .....	42
5 The correlation functions .....	46
6 Waiting times .....	49

## III STOCHASTIC PROCESSES

1 Definition .....	53
2 Stochastic processes in physics .....	56
3 Fourier transformation of stationary processes .....	60
4 The hierarchy of distribution functions .....	63
5 The vibrating string and random fields .....	67
6 Branching processes .....	72

## IV MARKOV PROCESSES

1 The Markov property .....	76
2 The Chapman–Kolmogorov equation .....	82

3	Stationary Markov processes . . . . .	85
4	The extraction of a subensemble . . . . .	91
5	Markov chains . . . . .	94
6	The decay process . . . . .	98

## V THE MASTER EQUATION

1	Derivation . . . . .	101
2	The class of $W$ -matrices . . . . .	106
3	The long time limit . . . . .	110
4	Closed isolated physical systems . . . . .	114
5	The increase of entropy . . . . .	118
6	Proof of detailed balance . . . . .	121
7	Expansion in eigenfunctions . . . . .	125
8	The macroscopic equation . . . . .	130
9	The adjoint equation . . . . .	135

## VI ONE-STEP PROCESSES

1	Definition; the Poisson process . . . . .	139
2	Random walk with continuous time . . . . .	142
3	General properties of one-step processes . . . . .	145
4	Examples of linear one-step processes . . . . .	149
5	Natural boundaries . . . . .	154
6	Solution of linear one-step processes with natural boundaries . . . . .	155
7	Artificial boundaries . . . . .	160
8	Artificial boundaries and normal modes . . . . .	164
9	Nonlinear one-step processes . . . . .	169
10	First-passage problems . . . . .	174

## VII CHEMICAL REACTIONS

1	Kinematics of chemical reactions . . . . .	180
2	Dynamics of chemical reactions . . . . .	185
3	The stationary solution . . . . .	188
4	Open systems . . . . .	191
5	Unimolecular reactions . . . . .	193
6	Collective systems . . . . .	198
7	Composite Markov processes . . . . .	203

## VIII THE FOKKER-PLANCK AND LANGEVIN EQUATIONS

1	Introduction . . . . .	209
2	Derivation of the Fokker-Planck equation . . . . .	214
3	Brownian motion . . . . .	217
4	The Rayleigh particle . . . . .	221
5	Application to one-step processes . . . . .	224
6	The multivariate linear Fokker-Planck equation . . . . .	228
7	Kramers' equation . . . . .	232
8	The Langevin approach . . . . .	237
9	Use and abuse of the Langevin approach . . . . .	246

## IX THE EXPANSION OF THE MASTER EQUATION

1	Introduction to the expansion . . . . .	253
2	General formulation of the expansion method . . . . .	257
3	The emergence of the macroscopic law . . . . .	263
4	The linear noise approximation . . . . .	267
5	Expansion of a multivariate master equation . . . . .	273
6	Higher orders . . . . .	278

## X THE DIFFUSION TYPE

1	Master equations of diffusion type . . . . .	284
2	Diffusion in an external field . . . . .	287
3	Diffusion in an inhomogeneous medium . . . . .	291
4	Multivariate diffusion equation . . . . .	293
5	The limit of zero fluctuations . . . . .	298

## XI UNSTABLE SYSTEMS

1	The bistable system . . . . .	304
2	The escape time . . . . .	312
3	Splitting probability . . . . .	316
4	The Malthus-Verhulst problem . . . . .	320
5	Critical fluctuations . . . . .	323
6	Diffusion in a bistable potential . . . . .	326
7	The parabolic approximation . . . . .	331
8	Limit cycles and fluctuations . . . . .	335
9	The laser as a diffusion problem . . . . .	338

## XII FLUCTUATIONS IN CONTINUOUS SYSTEMS

1 Introduction .....	345
2 Diffusion noise .....	347
3 The method of compounding moments .....	349
4 Fluctuations in phase space density .....	354
5 Fluctuations and the Boltzmann equation .....	358

## XIII THE STATISTICS OF JUMP EVENTS

1 Basic formulae and simple example .....	367
2 Jump events in nonlinear systems .....	370
3 Effect of incident photon statistics .....	372
4 Effect of incident photon statistics continued .....	376

## XIV STOCHASTIC DIFFERENTIAL EQUATIONS

1 Definitions .....	381
2 Heuristic treatment of multiplicative equations .....	384
3 The cumulant expansion .....	388
4 Three critical comments .....	394
5 Nonlinear stochastic differential equations .....	398
6 Long correlation times .....	404
7 The inhomogeneous linear equation .....	410
INDEX .....	413

