

## Contents

Preface	V
I Notations of Finite and Infinite Dimensional Manifold	1
II The Contact and Canonical Structure	
on the Cotangent Bundle	12
1. The Cotangent Bundle Contact Structure	12
2. The Cartan One-Form and the Legendre Transformation	19
3. The Canonical Structure on the Cotangent Bundle and	23
the Poisson Bracket	31
4. The Poisson Bracket and Hamilton's Equations	41
<ol> <li>The Contact Structure on Function Spaces</li> <li>The Canonical and Poisson-Bracket Structure on Function</li> </ol>	
Spaces	45
Spaces	
III Some Basic Theorems on Lie Algebra Theory	
1. The Burnside Theorems	55
2. Solvable Lie Algebras	62
3. The Cartan Criterion and the Killing Form	74
4. Lie Algebra Cohomology	81
5. Cohomology and Complete Reducibility	93
6. Deformations and Lie Algebra Cohomology	104
7. Lie Algebra Cohomology Relative to a Subalgebra	111
8. Abelian Extensions of Lie Algebras; The Levi Theorem	116
9. Lie Algebra Cohomology, the Filtered Lie Algebra	
Conjugacy Theorem, and The Malcev Conjugacy Theorem	123
10. Irreducible Lie Algebras of Linear Transformations	128
11. Central Extensions and Representations up to a Factor	131

IV Homology and Cohomology of Manifolds	
1. Introduction	140
2. Chain Complexes, Homology, and Cohomology	146
3. Cup-Products and Intersection Theory	158
<ol> <li>Invariance Under Homotopy and the Poincaré Lemma</li> <li>Relative Homology and Cohomology Defined by</li> </ol>	165
Submanifolds	170
6. De Rham's Theorem	173
V The Canonical Quantization Rules and Differential Operators	
1. Introduction	187
<ol> <li>Representations of the Poisson Bracket Lie Algebra as Homogeneous First Order Differential Operator</li> </ol>	191
3. Representations of the Poisson Bracket Lie Algebra by	1.71
First Order, Inhomogeneous Differential Operators	195
<ol> <li>Schrödinger Quantization by Differential Operators of General Dynamical Systems on Real Analytic Finite</li> </ol>	
Dimensional Manifolds	202
5. The Energy Operator as the Laplace-Beltrami Operator	212
<ol> <li>The Correspondence Principle for Riemannian Manifolds</li> <li>Quantization Problems Connected with the Hydrogen</li> </ol>	223
Atom and the Conformal Group	236
8. The Majorana Representation of the Lorentz Group in	
Terms of Poisson Brackets	247
VI The Symbol of a Linear Differential Operator and Its Physical Applications	
1. The Symbol of a Linear Differential Operator	257
2. The Symbol of the Laplace-Beltrami Operator	264
3. The Symbol of the Adjoint Operator	267
4. Intertwining Differential Operators on Homogeneous	
Vector Bundles and Zero-Mass Lorentz-Invariant Equations	271
5. The Symbol of Scalar Operators and the Schrödinger	
Quantization Rules	280
VII Further Work on the Transition	
Between Classical and Quantum Mechanics	
1. The Classical Limit of the Schrödinger Equation	295
2. Covariance of Quantum Mechanics Under Classical	19 <u>14</u>
Canonical Transformations	300
3. Replacement of the Hilbert Space of States of Particle	
Quantum Mechanics by an Infinite Dimensional	200
Canonical Manifold	308

Contents	ix
<ul><li>4. The Geometric Meaning of the Bohr-Sommerfeld Quantization Rules</li><li>5. The Higher Dimensional Generalization of the Bohr-Sommerfeld Quantization Rules</li></ul>	315 320
VIII Classical and Quantum Dynamical Systems of Homogeneous Spaces 1. The Rotating Top in Terms of Group Theory	326
<ol> <li>The Laplace-Beltrami Operator of Left-Invariant         Riemannian Metrics on Lie Groups     </li> <li>The Heisenberg Picture for Dynamical Systems on</li> </ol>	329
Finite Dimensional Homogeneous Manifolds  IX Invariance of the Schrödinger Equation  Under The Calilean Crown and Congrelizations	344
Under The Galilean Group and Generalizations  1. Multipliers for Intertwining Differential Operators	355
2. Study of the Ray-Representation Conditions	360
<ol> <li>Central Extensions of Groups</li> <li>Central Extensions and Multiplier Systems Acting on</li> </ol>	364
Vector Bundles	374
<ol> <li>Calculations for the Sc'rrödinger Operator</li> <li>Constant Coefficient Equations on R<sup>4</sup> Admitting Ray</li> </ol>	377
Representations and Their Study by Infinitesimal Methods	388
Bibliography	395