Contents

Preface **vii**

CHAPTER I General Concepts and Notations **1**
1. Spaces and Mappings 2
2. Fiber Mappings and Vector Bundles 4
3. Transformation Groups, Intertwining Maps, and Linear Group Actions on Vector Bundles 18
4. Differentiable Manifolds 26
5. Algebras and Modules 38
6. Differential Operators on Vector Bundles 43
7. Jet Bundles Associated with Modules 52

CHAPTER II Jet Bundles and the Classical Calculus of Variations **65**
1. Jet Bundles Associated with Fiber Spaces 65
2. Lagrangians and Fiber Spaces 70
3. Euler-Lagrange Operators on Vector Bundles and Associated Differential Operators 76
4. Symmetry Groups and Conserved Currents for General Lagrangians 80

CHAPTER III Connections in Vector Bundles and Yang-Mills Fields **87**
1. Linear Connections in Vector Bundles 88
2. The Fiber Space Whose Cross-Sections are Connections 98
3. Curvature as a Function on the Jet-Bundle 102
4. Deformation of Curvature and Linear Connections 103
5. Euler-Lagrange Operators Associated with the Curvature Tensor 107
6. The Principle of Minimal Interactions 110

7. Isomorphisms of Jet Vector Bundles and the Principle of Minimal Interaction 115

CHAPTER IV The Euler-Lagrange Operator in Terms of Differential Forms and Jet-Bundles 123
1. Differential-Geometric Notations 124
2. Prolongation of Bundle Isomorphisms and Vector Fields 131
3. The Calculus of Variations in Terms of Jet-Bundles 139
4. The Euler-Lagrange Operator in Terms of the Cartan Variational Formalism 148

CHAPTER V The Symplectic Structure on the Space of Extremals; Application to Quantum Field Theory 153
1. Introductory Remarks 153
2. The Symplectic Structure Associated with a Variational Problem 159
3. Conserved Currents Associated with Linear Differential Operations 167
4. The Symplectic Structure and Poisson Bracket for the Hamilton Equations 174
5. Vector Bundles Whose Fibers are Canonical Vector Spaces 179
6. Canonical Quantum Field Theory, in the Heisenberg Picture 184

CHAPTER VI Vector Bundles and Representations of Semidirect Product Groups 199
1. Vector Bundles Arising by Means of Semidirect Product Group Representations 200
2. Homogeneous Vector Bundles and Induced Representations 218
3. The Euclidean Motion Group 221
4. The Crystallographic Groups 232
5. Representations of the Poincaré Group 248

CHAPTER VII Poincaré-Invariant Differential Operators and Bundles 263
1. Poincaré Homogeneous Vector Bundles on R^4 264
2. Poincaré Invariant First Order Differential Operators 268
3. Finite Dimensional Linear Representations of the Lorentz Lie Algebra 275

4. Conditions for Invariant First Order Differential Operators on Poincaré-Homogeneous Vector Bundles 292
5. Mass and Spin Relations for Poincaré Invariant Differential Operators 307
6. A General Theorem About Finite Component Differential Operators 315

CHAPTER VIII Some General Properties of Poincaré Invariant Differential Operators **323**
1. Irreducible Differential Operators 324
2. General Remarks About Mass Spectrum of Finite Component Equations 335
3. Unitary Representations of the Poincaré Group Defined by Differential Operators 344
4. Conserved Currents Associated with General Differential Operators 353
5. Conserved Currents Associated with Lagrangians 359
6. Conserved Charge Density Currents Constructed as for the Dirac and Klein-Gordon Equations 363
7. The Charge-Conjugation Operation 372

CHAPTER IX Fock Space and Free Quantum Fields **379**
1. Symmetric and Skew-Symmetric Tensors 379
2. Hermitian-Symmetric Inner Products 384
3. Creation and Annihilation Operators 389
4. The Fock Space Construction of Free Quantum Fields 398
5. Annihilation and Creation Operators and the Heisenberg Lie Algebra 406
6. Clifford Algebras and Fermions 425

Bibliography **437**