



## TABLE OF CONTENTS

FOREWORD .....	7
CHAPTER I. GROUPS AND LIE ALGEBRAS .....	11
1. Some notation and terminology of group theory .....	11
2. Lie algebras .....	18
3. Heisenberg groups and Lie algebras .....	19
4. Analytic vectors of operators in Hilbert spaces .....	24
5. Holomorphic properties of the Fock space representation of the Heisenberg Lie algebra .....	28
6. Complexifications of groups and represen- tations .....	32
7. The representation of the complexified Heisenberg group .....	33
8. Representations defined using the universal enveloping algebra .....	36
CHAPTER II. GENERALIZED FUNCTIONS DEFINED BY ANALYTIC FUNCTIONS .....	41
1. Hilbert and Dirac spaces .....	41

2.	Generalized functions .....	43
3.	Generalized functions of one variable defined by holomorphic functions .....	45
4.	Representation of functions by a Cauchy integral formula .....	49
5.	Fourier transforms defined by Cauchy generalized functions .....	52
6.	The Mehler transform .....	54
7.	Generalized functions and measures defined by meromorphic functions .....	59
8.	The pointwise boundary values of holomor- phic functions .....	63
CHAPTER III. FORMAL ASPECTS OF RESOLVENT THEORY .....		71
1.	Introduction .....	71
2.	The resolvent of a Hermitian operator and the Weyl-Titchmarsh-Kodaira formula .....	72
3.	Resolvents associated with Fourier integrals .....	75
4.	Cauchy integral formulas for resolvents .....	79
CHAPTER IV. RESOLVENTS OF OPERATORS WITH DISCRETE SPECTRUM		81
1.	Introduction .....	81
2.	Resolvents of the Legendre differential operator in the "compact" region $-1 \leq t \leq 1$ and the Watson-Sommerfeld transform .....	85
CHAPTER V. LIMITS OF RESOLVENT EXPANSIONS ON LIE GROUPS .		97
1.	Introduction .....	97
2.	Limits of spectral representation .....	98
3.	Limits of subgroups and their Casimir operators .....	102
4.	Limiting relations for resolvents in terms of one-parameter unitary groups .....	109

5. Resolvents of $SO(3,R)$ and the Majorana representation of $SO(3,1)$ .....	111
CHAPTER VI. SOME GENERAL IDEAS ABOUT DEFORMATIONS OF GROUPS, SPACES AND FOURIER EXPANSIONS .....	
1. Introduction .....	115
2. Deformations of groups .....	115
3. Deformation of Fourier series into Fourier integrals .....	118
4. The deformation of $SO(2,R)$ to $R$ in terms of representations .....	124
5. A geometric point of view concerning deformations .....	129
6. The limit of Fourier series from the point of view of deformations of submanifolds .....	130
CHAPTER VII. HARMONIC ANALYSIS OF FUNCTIONS ON THE SPHERE 137	
1. Introduction .....	137
2. Spherical harmonics .....	139
3. Resolvents and spectral representations for functions on the sphere .....	144
4. The geometric interpretation of Legendre functions .....	149
5. The Mehler transform and the resolvent of the Laplace-Beltrami operator on $SO(2,1)/SO(2,R)$ .....	155
CHAPTER VIII. RESOLVENTS OF SEVERAL COMMUTING OPERATORS . 159	
1. Introduction .....	159
2. The resolvent of a finite number of commuting operators .....	159
3. Generalized eigenvector decompositions .....	160
4. Resolvents as integral operators .....	163

5. Resolvents of invariant differential operators on homogeneous spaces .....	166
6. The spectral measure and the Plancherel measure .....	169
CHAPTER IX. THE METHOD OF STATIONARY PHASE .....	179
1. Introduction .....	179
2. An iterative procedure for the asymptotic expansion .....	180
CHAPTER X. REPRESENTATIONS OF HEISENBERG GROUPS AND ALGEBRAS	
1. Introduction .....	185
2. The matrix elements of one-parameter subgroups of a Heisenberg group.....	185
3. Representations of Heisenberg algebras with a ground-state .....	189
4. The Lie algebra of currents .....	194
5. Quantization of linear mechanical systems .....	196
6. Quantization of degenerate Heisenberg algebras as operators in non-positive Hilbert spaces .....	202
CHAPTER XI. LINEAR QUANTUM FIELDS .....	207
1. Introduction .....	207
2. The Klein-Gordon field .....	209
3. Klein-Gordon fields on Riemannian manifolds.....	223
4. The Fock space realization of the free Lorentz invariant Klein-Gordon quantum field .....	230
5. Commutation relations of Klein-Gordon quantum fields on submanifolds .....	235
CHAPTER XII. THE GEOMETRY OF SCATTERING IN CLASSICAL PARTICLE MECHANICS .....	243
1. Introduction .....	243

2. The space of oriented lines of a finite dimensional vector space .....	244
3. The asymptotes of curves in V .....	246
4. Asymptotes for orbits of particles in classical mechanics .....	247
5. The scattering map .....	253
6. The Rutherford scattering formula for Coulomb scattering .....	255
7. The scattering cross-section .....	258
BIBLIOGRAPHY .....	263