

Contents

CHAPTER 1

Introduction: Differential Equations and Dynamical Systems	1
1.0. Existence and Uniqueness of Solutions	1
1.1. The Linear System $\dot{x} = Ax$	8
1.2. Flows and Invariant Subspaces	10
1.3. The Nonlinear System $\dot{x} = f(x)$	12
1.4. Linear and Nonlinear Maps	16
1.5. Closed Orbits, Poincaré Maps, and Forced Oscillations	22
1.6. Asymptotic Behavior	33
1.7. Equivalence Relations and Structural Stability	38
1.8. Two-Dimensional Flows	42
1.9. Peixoto's Theorem for Two-Dimensional Flows	60

CHAPTER 2

An Introduction to Chaos: Four Examples	66
2.1. Van der Pol's Equation	67
2.2. Duffing's Equation	82
2.3. The Lorenz Equations	92
2.4. The Dynamics of a Bouncing Ball	102
2.5. Conclusions: The Moral of the Tales	116

CHAPTER 3

Local Bifurcations	117
3.1. Bifurcation Problems	118
3.2. Center Manifolds	123
3.3. Normal Forms	138
3.4. Codimension One Bifurcations of Equilibria	145
3.5. Codimension One Bifurcations of Maps and Periodic Orbits	156

CHAPTER 4	
Averaging and Perturbation from a Geometric Viewpoint	166
4.1. Averaging and Poincaré Maps	167
4.2. Examples of Averaging	171
4.3. Averaging and Local Bifurcations	178
4.4. Averaging, Hamiltonian Systems, and Global Behavior: Cautionary Notes	180
4.5. Melnikov's Method: Perturbations of Planar Homoclinic Orbits	184
4.6. Melnikov's Method: Perturbations of Hamiltonian Systems and Subharmonic Orbits	193
4.7. Stability of Subharmonic Orbits	205
4.8. Two Degree of Freedom Hamiltonians and Area Preserving Maps of the Plane	212
CHAPTER 5	
Hyperbolic Sets, Symbolic Dynamics, and Strange Attractors	227
5.0. Introduction	227
5.1. The Smale Horseshoe: An Example of a Hyperbolic Limit Set	230
5.2. Invariant Sets and Hyperbolicity	235
5.3. Markov Partitions and Symbolic Dynamics	248
5.4. Strange Attractors and the Stability Dogma	255
5.5. Structurally Stable Attractors	259
5.6. One-Dimensional Evidence for Strange Attractors	268
5.7. The Geometric Lorenz Attractor	273
5.8. Statistical Properties: Dimension, Entropy, and Liapunov Exponents	280
CHAPTER 6	
Global Bifurcations	289
6.1. Saddle Connections	290
6.2. Rotation Numbers	295
6.3. Bifurcations of One-Dimensional Maps	306
6.4. The Lorenz Bifurcations	312
6.5. Homoclinic Orbits in Three-Dimensional Flows: Šilnikov's Example	318
6.6. Homoclinic Bifurcations of Periodic Orbits	325
6.7. Wild Hyperbolic Sets	331
6.8. Renormalization and Universality	342
CHAPTER 7	
Local Codimension Two Bifurcations of Flows	353
7.1. Degeneracy in Higher-Order Terms	354
7.2. A Note on k -Jets and Determinacy	360
7.3. The Double Zero Eigenvalue	364
7.4. A Pure Imaginary Pair and a Simple Zero Eigenvalue	376
7.5. Two Pure Imaginary Pairs of Eigenvalues without Resonance	397
7.6. Applications to Large Systems	413
APPENDIX	
Suggestions for Further Reading	423
Glossary	427
References	433
Index	449

