



## Contents

	page
<b>Chapter 1. INTRODUCTION . . . . .</b>	<b>1</b>
1.1 The notion of stability . . . . .	1
1.2 The nature of non-linear stability problems . . . . .	4
1.3 Formal approach to stability theory . . . . .	9
<b>Chapter 2. A CLASS OF PROBLEMS IN ONE-DIMENSIONAL SPACE . . . . .</b>	<b>11</b>
2.1 Preliminary remarks . . . . .	11
2.2 Formulation . . . . .	11
2.3 Behaviour and properties of the linearized solutions . . . . .	13
2.4 Series expansion in the case of self-adjoint operators. . . . .	15
2.5 Series expansion in the case of not self-adjoint operators . . . . .	16
2.6 Interpretation of the series expansion in terms of the GREEN's function	17
2.7 The system of equations for the amplitude-functions . . . . .	18
<b>Chapter 3. BEHAVIOUR OF SOLUTIONS . . . . .</b>	<b>19</b>
3.1 Formal simplification of the system of equations . . . . .	19
3.2 Stable and unstable stationary solutions . . . . .	20
3.3 Effects of interactions. Forced solutions. . . . .	23
3.4 Analysis of forced solutions . . . . .	24
3.5 Instability to finite size perturbations. . . . .	26
3.6 Other types of behaviour . . . . .	28
<b>Chapter 4. ASYMPTOTIC METHODS FOR PROBLEMS IN ONE-DIMENSIONAL SPACE</b>	<b>28</b>
4.1 General outline . . . . .	28
4.2 Weak stability or instability: the case $\alpha_{ij}^{(n)} = 0$ . . . . .	32
4.3 Weak stability or instability: the case $\alpha_{00}^{(0)} = 0$ . . . . .	34
4.4 Weak stability and instability: the case $\alpha_{00}^{(0)} \neq 0$ . . . . .	35
4.5 Method of approximation for the case of simple developed instability	38
4.6 Behaviour of solutions as functions of time . . . . .	41
<b>Chapter 5. ANALYSIS OF SOME ONE-DIMENSIONAL PROBLEMS . . . . .</b>	<b>43</b>
5.1 Introductory remarks . . . . .	43
5.2 BURGERS' mathematical model of turbulence . . . . .	43
5.3 Modification of BURGERS' model. The problem of stability . . . . .	45
5.4 Asymptotic expansions in BURGERS' model . . . . .	46
5.5 Another simple mathematical model . . . . .	49
<b>Chapter 6. A CLASS OF PROBLEMS IN TWO-DIMENSIONAL SPACE . . . . .</b>	<b>52</b>
6.1 Introductory remarks . . . . .	52
6.2 Formulation . . . . .	52

	page
6.3 The problem of stability. Linearized theory . . . . .	54
6.4 Fourier-analysis of the non-linear stability problem . . . . .	56
6.5 Orthogonality relations . . . . .	58
6.6 Initial conditions. . . . .	61
 Chapter 7. ASYMPTOTIC THEORY OF PERIODIC SOLUTIONS. . . . .	63
7.1 Basic equations and transformations. . . . .	63
7.2 Forced solutions for the components $\psi_m, m \neq 1$ . . . . .	64
7.3 Analysis of the component $\psi_1$ . . . . .	66
7.4 Further analysis of the forced solutions for $\psi_m, m \neq 1$ . . . . .	67
7.5 The equations of the asymptotic approximation . . . . .	69
7.6 Harmonic solutions. . . . .	71
7.7 A simple example . . . . .	73
 Chapter 8. STABILITY OF PERIODIC SOLUTIONS. . . . .	77
8.1 Introduction . . . . .	77
8.2 Formulation of the stability problem . . . . .	78
8.3 Analysis of small parameters . . . . .	81
8.4 Perturbations in the region $\tau_0^{(k)} = o(1)$ . . . . .	83
8.5 Perturbations in the region $\tau_0^{(k)} = o(\varepsilon^2)$ . . . . .	84
8.6 Reduction of the system of equations . . . . .	85
8.7 Forced solutions for $\psi_{\varepsilon\sigma}$ and $\psi_{2k_0 \pm \varepsilon\sigma}$ . . . . .	87
8.8 The equations for $A_0^{(k)}$ and $A_0^{(k, \prime)}$ . . . . .	89
8.9 Solution of the stability problem for $k_0 = k_{cr}$ . . . . .	90
8.10 Regions of validity of the asymptotic results . . . . .	92
8.11 Stability of periodic solutions in the case $k_0 \neq k_{cr}$ . . . . .	93
8.12 Summary and interpretation of results. . . . .	95
 Chapter 9. PERIODIC SOLUTIONS IN POISEUILLE FLOW . . . . .	97
9.1 Introduction. . . . .	97
9.2 Formulation of the stability problem . . . . .	99
9.3 Linearized stability theory . . . . .	100
9.4 The adjoint linearized problem . . . . .	106
9.5 Periodic solutions. . . . .	109
9.6 Discussion of the results . . . . .	111
 BIBLIOGRAPHY . . . . .	114
INDEX	116

