## Contents

Part I	
Ergodicity and Mixing. Examples of Dynamic Systems	1
Chapter 1	
Basic Definitions of Ergodic Theory	3
§1. Definition of Dynamical Systems	3
§2. The Birkhoff-Khinchin Ergodic Theorem. Ergodicity	11
§3. Nonergodic Systems. Decomposition into Ergodic Components	16
§4. Averaging in the Ergodic Case	18
§5. Integral and Induced Automorphisms	20 22
§6. Weak Mixing, Mixing, Multiple Mixing §7. Unitary and Isometric Operators Adjoint to Dynamical Systems	26
§8. Dynamical Systems on Compact Metric Spaces	36
Chapter 2	
Smooth Dynamical Systems on Smooth Manifolds	43
§1. Invariant Measures Compatible with Differentiability	43
§2. Liouville's Theorem and the Dynamical Systems of Classical Mechanics	47
§3. Integrable Dynamical Systems	58
Chapter 3	
Smooth Dynamical Systems on the Torus	64
§1. Translations on the Torus	64
§2. The Lagrange Problem	69
§3. Homeomorphisms of the Circle	73
§4. The Denjoy Theorem	83 87
§5. Arnold's Example  S6. The Freedicity of Diffeomorphisms of the Circle with	0/
§6. The Ergodicity of Diffeomorphisms of the Circle with Respect to Lebesgue Measure	92
Respect to Leoesgue Measure	

Contents

Chapter 4	0.7
Dynamical Systems of Algebraic Origin	96
§1. Translations on Compact Topological Groups §2. Skew Translations and Compound Skew Translations on	96
Commutative Compact Groups	100
§3. Endomorphisms and Automorphisms of Commutative Compact Groups	104
§4. Dynamical Systems on Homogenous Spaces of the Group SL(2, ℝ)	112
Chapter 5	
Interval Exchange Transformations	122
§1. Definition of Interval Exchange Transformations	122
§2. An Estimate of the Number of Invariant Measures	124
§3. Absence of Mixing	127
§4. An Example of a Minimal but not Uniquely Ergodic Interval Exchange Transformation	132
Chapter 6 Billiards	138
§1. The Construction of Dynamical Systems of the Billiards Type	138
§2. Billiards in Polygons and Polyhedra	143
§3. Billiards in Domains with Convex Boundary	149
§4. Systems of One-dimensional Point-like Particles	152
§5. Lorentz Gas and Systems of Hard Spheres	154
Chapter 7	
Dynamical Systems in Number Theory	157
§1. Uniform Distribution	157
§2. Uniform Distribution of Fractional Parts of Polynomials	159
§3. Uniform Distribution of Fractional Parts of Exponential Functions §4. Ergodic Properties of Decompositions into Continuous Fractions and	164
Piecewise-monotonic Maps	165
Chanter 8	
Chapter 8  Dynamical Systems in Probability Theory	178
§1. Stationary Random Processes and Dynamical Systems	178
§2. Gauss Dynamical Systems	188
Chapter 9	
Examples of Infinite Dimensional Dynamical Systems	193
§1. Ideal Gas	193
§2. Dynamical Systems of Statistical Mechanics	200
§3. Dynamical Systems and Partial Differential Equations	223

Contents	ix
Part II	
Basic Constructions of Ergodic Theory	225
Chapter 10	
Simplest General Constructions and Elements of Entropy	225
Theory of Dynamical Systems	227
§1. Direct and Skew Products of Dynamical Systems §2. Metric Isomorphism of Skew Products. Equivalence of Dynamical	227
Systems in the Sense of Kakutani §3. Time Change in Flows	233 235
§4. Endomorphisms and Their Natural Extensions	239
§5. The Rohlin-Halmos Lemma	242
§6. Entropy	246
§7. Metric Isomorphism of Bernoulli Automorphisms	258
§8. K-systems and Exact Endomorphisms	280
Chapter 11	
Special Representations of Flows	292
§1. Definition of Special Flows §2. Statement of the Main Theorem on Special Representation of Flows	292
and Examples of Special Flows	295
§3. Proof of the Theorem on Special Representation §4. Rudolph's Theorem	300 309
y4. Rudolphi 3 Theorem	507
Part III	
Spectral Theory of Dynamical Systems	323
Chapter 12	
Dynamical Systems with Pure Point Spectrum	325
§1. General Properties of Eigen-Values and Eigen-Functions of	
Dynamical Systems	325
§2. Dynamical Systems with Pure Point Spectrum. The Case of Discrete Time	328
§3. Dynamical Systems with Pure Point Spectrum. The Case of Continuous Time	334
Chapter 13	
Examples of Spectral Analysis of Dynamical Systems	338
§1. Spectra of K-automorphisms	338
§2. Spectra of Ergodic Automorphisms of Commutative Compact Groups	340
§3. Spectra of Compound Skew Translations on the Torus and of Their	
Perturbations	342
§4. Examples of the Spectral Analysis of Automorphisms with Singular	247
Spectrum §5. Spectra of K-flows	347 353
oF	

V	-
Λ	

Index

483

Chapter 14	
Spectral Analysis of Gauss Dynamical Systems	356
§1. The Decomposition of Hilbert Space $L^2(M, \mathfrak{S}, \mu)$ into Hermite-Ito	
Polynomial Subspaces	356
§2. Ergodicity and Mixing Criteria for Gauss Dynamical Systems	368
§3. The Maximal Spectral Type of Unitary Operators Adjoint to Gauss Dynamical Systems	271
§4. Gauss Dynamical Systems with Simple Continuous Spectrum	371
§5. Gauss Dynamical Systems with Finite Multiplicity Spectrum	379
Part IV	
Approximation Theory of Dynamical Systems by Periodic Dynamical	
Systems and Some of its Applications	387
Chapter 15	
Approximations of Dynamical Systems	389
§1. Definition and Types of Approximations. Ergodicity and Mixing Conditions	389
§2. Approximations and Spectra	395
§3. An Application of Approximation Theory: an Example of an Ergodic	
Automorphism with a Spectrum Lacking the Group Property	398
§4. Approximation of Flows	404
Chapter 16	
Special Representations and Approximations of Smooth Dynamical	
Systems on the Two-dimensional Torus	408
§1. Special Representations of Flows on the Torus	408
§2. Dynamical Systems with Pure Point Spectrum on the Two-dimensional	
Torus	418
§3. Approximations of Flows on the Torus	425
§4. Example of a Smooth Flow with Continuous Spectrum on the Two-dimensional Torus	434
I WO-diliterisional Torus	TOT
Appendix 1	
Lebesgue Spaces and Measurable Partitions	449
Appendix 2	
Relevant Facts from the Spectral Theory of Unitary Operators	453
Appendix 3	
Proof of the Birkhoff-Khinchin Theorem	459
Appendix 4	
Kronecker Sets	463
Bibliographical Notes	467
Bibliography	475
Dionography	713