Contents

1

Introduction 1

- 1.1 Ordering 1
- 1.2 Asymptotic sequences and expansions 2

2

Limit process expansions applied to ordinary differential equations 4

- 2.1 Linear oscillator: regular problem 4
- 2.2 Linear oscillator: singular problem 7
- 2.3 Singular perturbation problem with variable coefficients 13
- 2.4 Theorem of A. Erdelyi 26
- 2.5 Model nonlinear example for singular perturbations 29
- 2.6 Relaxation oscillations of the van der Pol oscillator 38
- 2.7 Singular boundary problems 55
- 2.8 Higher-order example: beam string 69
- 2.9 Extension theorem of S. Kaplun 77

Two-variable expansion procedures 79

- 3.1 Poincaré method; periodic solution 82
- 3.2 Linear oscillator with small damping 87
- 3.3 Oscillator with cubic damping 91
- 3.4 Approach to limit cycle of van der Pol oscillator 93
- 3.5 Mathieu equation 94
- 3.6 Adiabatic invariance 102
- 8.7 Sturm-Liouville equation; differential equation with a large parameter 105
- 3.8 Nonlinear oscillations with slowly varying coefficients 111

4

Applications to partial differential equations 120

- 4.1 Singular perturbation problems for second-order partial differential equations 120
- 4.2 Boundary layer theory in viscous incompressible flow 142
- 4.3 Magnetohydrodynamic pipe flow 162
- 4.4 Viscous boundary layer in a rotating fluid 176
- 4.5 Singular boundary-value problems 181

5

Derivation of approximate equations. Several parameters. Similarity laws 222

- 5.1 Expansion procedures of thin airfoil theory at various Mach numbers 222
- 5.2 Near-field and far-field equations 234

INDEX 257