## Contents

1.	INTRODUCTION		1
	1.1	Ordinary Differential Equations	2
	1.2	Historical Remarks	5
2.	FIRST	ORDER DIFFERENTIAL EQUATIONS	10
	2.1	Linear Equations	10
	2.2	Further Discussion of Linear Equations	17
	2.3	Nonlinear Equations	21
	2.4	Separable Equations	29
	2.5	Exact Equations	34
	2.6	Integrating Factors	39
304	2.7	Homogeneous Equations	43
	2.8	Miscellaneous Problems	47
	2.9	Applications of First Order Equations	51
	2.10	Elementary Mechanics	61
	*2.11	The Existence and Uniqueness Theorem	69
	*2.12	The Existence Theorem from a More Modern Viewpoint	80
		Appendix. Derivation of Equation of Motion of Body with Variable Mass	82
3.	SECO	ND ORDER LINEAR EQUATIONS	85
	3.1	Introduction	85
	3.2	Fundamental Solutions of the Homogeneous Equation	90
	3.3	Linear Independence	99
			~

5	127
XII	Contents
VII	Contonis

	3.4	Reduction of Order	103
	3.5	Homogeneous Equations with Constant Coefficients	106
	3.5.1	Complex Roots	110
	3.6	The Nonhomogeneous Problem	114
	3.6.1	The Method of Undetermined Coefficients	117
	3.6.2	The Method of Variation of Parameters	124
	3.7	Mechanical Vibrations	129
	3.7.1	Free Vibrations	133
	3.7.2	Forced Vibrations	138
	3.8	Electrical Networks	142
4.		S SOLUTIONS OF SECOND ORDER LINEAR TIONS	148
	4.1	Introduction. Review of Power Series	148
	4.2	Series Solutions near an Ordinary Point, Part I	152
	4.2.1	Series Solutions near an Ordinary Point, Part II	159
	4.3	Regular Singular Points	167
	4.4	Euler Equations	171
	4.5	Series Solutions near a Regular Singular Point, Part I	177
	4.5.1	Series Solutions near a Regular Singular Point, Part II	183
	*4.6	Series Solutions near a Regular Singular Point; $r_1 = r_2$ and $r_1 - r_2 = N$	188
	*4.7	Bessel's Equation	191
5.	HIGH	ER ORDER LINEAR EQUATIONS	202
	5.1	Introduction	202
	5.2	General Theory of nth Order Linear Equations	204
	5.3	The Homogeneous Equation with Constant Coefficients	208
	5.4	The Method of Undetermined Coefficients	215
	5.5	The Method of Variation of Parameters	218

		Contents	xiii
6.	THE I	LAPLACE TRANSFORM	222
	6.1	Introduction. Definition of the Laplace Transform	222
	6.2	Solution of Initial Value Problems	228
	6.3	Step Functions	237
	6.3.1	A Differential Equation with a Discontinuous Forcing Function	245
	6.4	Impulse Functions	249
	6.5	The Convolution Integral	253
	6.6	General Discussion and Summary	258
7.	SYSTE	MS OF FIRST ORDER EQUATIONS	261
	7.1	Introduction	261
	7.2	Solution of Linear Systems by Elimination	266
	7.3	Review of Matrices	272
	7.4	Basic Theory of Systems of First Order Linear Equations	282
	7.5	Linear Homogeneous Systems with Constant Coefficients	288
	7.6	Inverses, Eigenvalues, and Eigenvectors	294
	7.7	Fundamental Matrices	300
	7.8	Complex Roots	307
	7.9	Repeated Roots	312
	7.10	Nonhomogeneous Linear Systems	317
	7.11	The Laplace Transform for Systems of Equations	322
8.	NUME	RICAL METHODS	328
	8.1	Introduction	328
	8.2	The Euler or Tangent Line Method	330
	8.3	The Error	338
	8.4	An Improved Euler Method	345
	8.5	The Three-Term Taylor Series Method	350
	8.6	The Runge-Kutta Method	353
	8.7	Some Difficulties with Numerical Methods	357
	8.8	A Multistep Method	361
	8.9	Systems of First Order Equations	365

9.	NONI	INEAR DIFFERENTIAL EQUATIONS	
	AND	STABILITY	369
	9.1	Introduction	369
	9.2	Solutions and Trajectories	375
	9.3	The Phase Plane; The Linear System	382
	9.4	Stability; Almost Linear Systems	392
	9.5	Liapounov's Second Method	403
	*9.6	Periodic Solutions and Limit Cycles	413
10.		TIAL DIFFERENTIAL EQUATIONS FOURIER SERIES	422
	10.1	Introduction	422
	10.2	Heat Conduction and Separation of Variables	423
	10.3	Fourier Series	431
	10.4	The Fourier Theorem	438
	10.5	Even and Odd Functions	445
	10.6	Solution of Other Heat Conduction Problems	453
	10.7	The Wave Equation; Vibrations of an Elastic String	460
	10.8	Laplace's Equation	470
		Appendix A. Derivation of the Heat Conduction Equation	478
		Appendix B. Derivation of the Wave Equation	481
11.		NDARY VALUE PROBLEMS AND RM-LIOUVILLE THEORY	485
	11.1	Introduction	485
	11.2	Linear Homogeneous Boundary Value Problems: Eigenvalues and Eigenfunctions	486
	11.3	Sturm-Liouville Boundary Value Problems	493
	11.4	Solution of Nonhomogeneous Boundary Value Problems	501
	*11.5	Singular Sturm-Liouville Problems	511
	*11.6	Further Remarks on the Method of Separation of Variables; A Bessel Series Expansion	518
	*11.7	Series of Orthogonal Functions; Mean Convergence	525
	ANS	WERS TO PROBLEMS	A-1
	IND	EX	I-1

