Contents

Notation X

Introduction 1

I. Dynamical Systems 5

1. Definition and Related Notation 5
2. Examples of Dynamical Systems 6
Notes and References 10

II. Elementary Concepts 12

1. Invariant Sets and Trajectories. 12
2. Critical Points and Periodic Points 15
3. Trajectory Closures and Limit Sets 19
4. The First Prolongation and the Prolongational Limit Set 24
Notes and References 30

III. Recursive Concepts. 31

1. Definition of Recursiveness 31
2. Poisson Stable and Non-wandering Points 31
3. Minimal Sets and Recurrent Points 36
4. Lagrange Stability and Existence of Minimal Sets 41
Notes and References 42

IV. Dispersive Concepts 43

1. Unstable and Dispersive Dynamical Systems 43
2. Parallelizable Dynamical Systems 48
Notes and References 55

V. Stability Theory. 56

1. Stability and Attraction for Compact Sets 56
2. Liapunov Functions: Characterization of Asymptotic Stability . . . 66
3. Topological Properties of Regions of Attraction 79
4. Stability and Asymptotic Stability of Closed Sets 84
5. Relative Stability Properties 99
6. Stability of a Motion and Almost Periodic Motions 106
Notes and References 111

VI. Flow near a Compact Invariant Set. 114

1. Description of Flow near a Compact Invariant Set 114
2. Flow near a Compact Invariant Set (Continued) 116
Notes and References 117

VII. Higher Prolongations 119

1. Definition of Higher Prolongations 120
2. Absolute Stability 124
3. Generalized Recurrence 129
Notes and References 133

VIII. $\mathscr{C}^1$-Liapunov Functions for Ordinary Differential Equations 134

1. Introduction 134
2. Preliminary Definitions and Properties 136
3. Local Theorems 138
4. Extension Theorems 145
5. The Structure of Liapunov Functions 150
6. Theorems Requiring Semidefinite Derivatives 156
7. On the Use of Higher Derivatives of a Liapunov Function 160
Notes and References 162

IX. Non-continuous Liapunov Functions for Ordinary Differential Equations 166

1. Introduction 166
2. A Characterization of Weak Attractors 169
3. Piecewise Differentiable Liapunov Functions 172
4. Local Results 176
5. Extension Theorems 177
6. Non-continuous Liapunov Functions on the Region of Weak Attraction 179
Notes and References 183

References 185

Author Index 221

Subject Index 223