

Contents

Preface

Chapter 1 Introduction

1.1 General Remarks 1

1.2 Notation and Mathematical Background 6

1.3 Unconstrained Minimization 18

1.3.1 Convergence Analysis of Gradient Methods 20

1.3.2 Steepest Descent and Scaling 39

1.3.3 Newton's Method and Its Modifications 40

1.3.4 Conjugate Direction and Conjugate Gradient Methods 49

1.3.5 Quasi-Newton Methods 59

1.3.6 Methods Not Requiring Evaluation of Derivatives 65

1.4 Constrained Minimization 66

1.5 Algorithms for Minimization Subject to Simple Constraints 76

1.6 Notes and Sources 93

Chapter 2 The Method of Multipliers for Equality

Constrained Problems

2.1 The Quadratic Penalty Function Method 96

2.2 The Original Method of Multipliers 104

2.2.1 Geometric Interpretation 105

2.2.2 Existence of Local Minima of the Augmented Lagrangian 107

2.2.3 The Primal Functional 113

2.2.4 Convergence Analysis 115

2.2.5 Comparison with the Penalty Method Computational Aspects 121

2.3 Duality Framework for the Method of Multipliers 125

2.3.1 Step Size Analysis for the Method of Multipliers 126

2.3.2 The Second-Order Multiplier Iteration 133

2.3.3 Quasi-Newton Versions of the Second-Order Iteration 138

2.3.4 Geometric Interpretation of the Second-Order Multiplier Iteration 139

2.4 Multiplier Methods with Partial Elimination of Constraints	141
2.5 Asymptotically Exact Minimization in Methods of Multipliers	147
2.6 Primal-Dual Methods Not Utilizing a Penalty Function	153
2.7 Notes and Sources	156
Chapter 3 The Method of Multipliers for Inequality Constrained and Nondifferentiable Optimization Problems	
3.1 One-Sided Inequality Constraints	158
3.2 Two-Sided Inequality Constraints	164
3.3 Approximation Procedures for Nondifferentiable and Ill-Conditioned Optimization Problems	167
3.4 Notes and Sources	178
Chapter 4 Exact Penalty Methods and Lagrangian Methods	
4.1 Nondifferentiable Exact Penalty Functions	180
4.2 Linearization Algorithms Based on Nondifferentiable Exact Penalty Functions	196
4.2.1 Algorithms for Minimax Problems	196
4.2.2 Algorithms for Constrained Optimization Problems	201
4.3 Differentiable Exact Penalty Functions	206
4.3.1 Exact Penalty Functions Depending on x and X	206
4.3.2 Exact Penalty Functions Depending Only on x	215
4.3.3 Algorithms Based on Differentiable Exact Penalty Functions	217
4.4 Lagrangian Methods Local Convergence	231
4.4.1 First-Order Methods	232
4.4.2 Newton-like Methods for Equality Constraints	234
4.4.3 Newton-like Methods for Inequality Constraints	248
4.4.4 Quasi-Newton Versions	256
4.5 Lagrangian Methods Global Convergence	257
4.5.1 Combinations with Penalty and Multiplier Methods	258
4.5.2 Combinations with Differentiable Exact Penalty Methods - Newton and Quasi-Newton Versions	260
4.5.3 Combinations with Nondifferentiable Exact Penalty Methods - Powell's Variable Metric Approach	284
4.6 Notes and Sources	297
Chapter 5 Nonquadratic Penalty Functions Convex Programming	
5.1 Classes of Penalty Functions and Corresponding Methods of Multipliers	302
5.1.1 Penalty Functions for Equality Constraints	303
5.1.2 Penalty Functions for Inequality Constraints	305
5.1.3 Approximation Procedures Based on Nonquadratic Penalty Functions	312
5.2 Convex Programming and Duality	315
5.3 Convergence Analysis of Multiplier Methods	326
5.4 Rate of Convergence Analysis	341
5.5 Conditions for Penalty Methods to Be Exact	359

5.6 Large Scale Separable Integer Programming Problems and the Exponential Method of Multipliers	364
5.6.1 An Estimate of the Duality Gap	371
5.6.2 Solution of the Dual and Relaxed Problems	376
5.7 Notes and Sources	380
References	383
Index	393