Contents

Foreword vii
Preface ix
Introduction 1

PART I. MATHEMATICAL MODELS

1 Linear and Nonlinear Oscillations BY SOLOMON LEFSCHETZ 7

1.1 Introduction *7*
1.2 Harmonic Oscillators *7*
1.3 Damped Oscillations *10*
1.4 Forced Oscillations *11*
1.5 Linear and Nonlinear Systems *13*
1.6 Certain Nonlinear Systems *15*
1.7 Nonlinear Oscillations in Conservative Systems *17*
1.8 Nonlinear Forced Oscillations *18*
1.9 Multivibrator Circuits *20*
1.10 Mathematical Treatment of Nonlinear Problems *21*
1.11 Methods of Approximation *25*
1.12 Duffing's Method *26*
1.13 Poincaré's Perturbation Method *28*

2 Equilibrium Analysis: The Stability Theory of Poincaré and Liapunov BY RICHARD BELLMAN 30

2.1 Introduction *30*
2.2 The Stability Theory of Poincaré and Liapunov *31*
2.3 Stability Theory of Linear Equations *33*
2.4 Differential-difference Equations *34*
2.5 The Heat Equation *34*

3 Exterior Ballistics BY JOHN W. GREEN 36

3.1 Introduction *36*
3.2 Selection of Coordinate Systems *37*

3.3 Aerodynamic Forces on a Projectile *41*
3.4 The Equations of Motion *47*
3.5 Ballistic and Firing Tables *48*
3.6 Corrections for Small Effects *50*
3.7 Bombing from Airplanes *54*
3.8 Effects of Aerodynamic Forces Other than Drag *56*
3.9 Conclusion and References *57*

4 Elements of the Calculus of Variations
BY MAGNUS R. HESTENES **59**

4.1 Introduction *59*
4.2 Some Elementary Variational Problems *60*
4.3 General Statements of Problems; Necessary Conditions for a Minimum *64*
4.4 Derivation of the Euler Equations *67*
4.5 Special Cases *70*
4.6 Integrands of the Form $f(x,y)$ *74*
4.7 Hamilton's Principle *75*
4.8 Hamiltonians *78*
4.9 Isoperimetric Problems *80*
4.10 Variable End-point Problems *83*
4.11 Minima of Functions of Integrals *85*
4.12 Problem of Bolza *87*
4.13 Multiple-integral Problems *89*

5 Hyperbolic Partial Differential Equations and Applications BY RICHARD COURANT **92**

5.1 Introduction *92*
5.2 Relation between Partial Differential Equations and Reality *93*
5.3 Statistical Processes and Partial Differential Equations *94*
5.4 Classification of Linear Partial Differential Equations; Plane Waves *96*
5.5 Initial-value Problem for the Wave Equation *98*
5.6 Nonlinear Hyperbolic Equations *101*
5.7 Finite-difference Methods *105*

6 Boundary-value Problems in Elliptic Partial Differential Equations BY MENAHEM M. SCHIFFER **110**

6.1 What Is a Properly Posed Problem in Partial Differential Equations? *110*

6.2 Theory of Heat Conduction; the Three Main Boundary-value Problems *112*
6.3 Fundamental Singularities and Green's Functions *116*
6.4 Maximum Principle, Kernel Function, and Dirichlet Integral *122*
6.5 Illustrations from Fluid Dynamics and Electrostatics *128*
6.6 Variation of the Green's Functions with the Domain *134*
6.7 Variation of the Green's Functions with the Coefficients of the Differential Equation *140*

7 The Elastostatic Boundary-value Problems BY IVAN S. SOKOLNIKOFF **145**

Formulation of Problems

7.1 Introduction *145*
7.2 Two Basic Types of Problems *146*
7.3 Characterization of Displacements; Strain *146*
7.4 Characterization of the State of Stress *147*
7.5 Conditions of Equilibrium *148*
7.6 Two- and Three-dimensional Problems *149*
7.7 The Cylinder; Forces Constant along the Length *150*
7.8 The Cylinder; Forces Distributed over the Ends *152*
7.9 The Cylinder; Combined Problem *153*

A Method of Solution

7.10 Computational Considerations *153*
7.11 Formulation in Terms of Complex-variable Theory *154*
7.12 The Dirichlet Problem *157*
7.13 The Biharmonic Boundary-value Problem *159*
7.14 Conclusion *160*

PART 2. PROBABILISTIC PROBLEMS

8 The Theory of Prediction BY NORBERT WIENER **165**

8.1 Time Series and Causality *165*
8.2 The Ergodic Theorem *165*
8.3 Time Series *171*
8.4 The Prediction of an Incompletely Given Message *180*
8.5 Continuous Prediction *182*
8.6 Multiple Prediction and Causality *184*

9 The Theory of Games BY H. FREDERIC BOHNENBLUST **191**

9.1 Introduction *191*

9.2 Scope of the Theory *192*
9.3 Optimal Pure Strategies *194*
9.4 An Example in Planning *195*
9.5 Mixed Strategies *197*
9.6 Symmetry; an Example of Tactical Timing *199*
9.7 The max-min = min-max Relation *202*
9.8 An Application to Economics *204*
9.9 Successive Approximations *206*
9.10 The General Notion of Strategy; a Pokerlike Game *207*
9.11 Conclusion *209*

10 Applied Mathematics in Operations Research **211**
BY GILBERT W. KING

Definition

10.1 The Scientific Method *211*
10.2 Models *211*
10.3 Prediction *212*

Deterministic Models

10.4 Inviolable Solutions *213*
10.5 Linear Inequalities *213*
10.6 Duality *216*
10.7 Production Scheduling *217*
10.8 Solution of the Dual *219*
10.9 Nonlinear Forms *220*
10.10 The Lagrangian *222*
10.11 Game Theory *224*

Probabilistic Models

10.12 Stochastic Variation *227*
10.13 The Inventory Problem *227*
10.14 Monte Carlo Methods *229*
10.15 Random Selection *232*
10.16 Customers *233*
10.17 Theory of Prediction *238*
10.18 Markoff Processes *240*
10.19 Conclusion *242*

11 The Theory of Dynamic Programming **243**
BY RICHARD BELLMAN

11.1 Introduction *243*

11.2 Preliminary Discussion *244*

Optimal Allocation of Resources

11.3 Introduction *246*
11.4 A Purchasing Problem *246*
11.5 Classical Formulation *247*
11.6 The Dynamic-programming Formulation *247*
11.7 Infinite-stage Approximation *248*
11.8 An Existence and Uniqueness Theorem *249*
11.9 Analytic Results *251*
11.10 Computational Techniques *252*
11.11 Uncertainty *254*

The Gold-mining Equation

11.12 Introduction *256*
11.13 The Gold-mining Problem *256*
11.14 Mathematical Formulation *257*
11.15 Existence and Uniqueness *257*
11.16 Solution *258*
11.17 Interpretation of the Solution *259*
11.18 Nonlinear Utility *260*
11.19 A Continuous Version *261*
11.20 Derivation of the Differential Equations *263*
11.21 The Solution *264*

Bottleneck Problems

11.22 Introduction *265*
11.23 A Typical Problem *265*
11.24 A Continuous Version *267*
11.25 Some Remarks on Notation *268*
11.26 The Basic Functional Equation *269*
11.27 Infinitesimal Analogue *270*
11.28 A Dual Problem *271*
11.29 More General Problems *273*
11.30 Control Problems *273*

Games of Survival

11.31 Introduction *274*
11.32 Games of Survival *274*
11.33 Games of Survival—Nonzero-sum *275*

12 Monte Carlo Methods BY GEORGE W. BROWN **279**

Introduction

12.1 The Nature of Monte Carlo *279*

12.2 Basic Statistical Concepts *280*

Monte Carlo Evaluation of Integrals

12.3 Random Sampling as a Method of Calculation *283*
12.4 A Special Case *285*
12.5 A Trivial Numerical Example *289*
12.6 Applications Originating as Multiple Integrals *290*

Monte Carlo Methods Related to Diffusion Processes

12.7 Uniform Random Walk in Two Dimensions *291*
12.8 Laplace's Equation *293*
12.9 Generalizations *297*

Methods of Sampling from Specified Distributions

12.10 Sources of Random Numbers *300*
12.11 Transformations of Random Numbers *301*

PART 3. COMPUTATIONAL CONSIDERATIONS

13 Matrices in Engineering BY LOUIS A. PIPES **307**

13.1 Introduction *307*

The Fundamentals of Matrix Algebra

13.2 Definition of a Matrix *308*
13.3 Principal Types of Matrices *308*
13.4 Definitions and Fundamental Algebraic Operations *309*
13.5 Multiplication of Matrices *309*
13.6 The Inverse Matrix; Matrix Division *312*
13.7 Partitioning of Matrices into Submatrices *312*
13.8 Characteristic Matrix and Characteristic Equation of a Matrix *314*
13.9 Reduction of a Matrix to Diagonal Form *314*
13.10 The Trace of a Matrix *316*
13.11 The Cayley-Hamilton Theorem *317*
13.12 Solution of Linear Equations and Inversion of Large Matrices *318*

The Calculus of Matrices

13.13 Differentiation of a Matrix *321*
13.14 Integration of a Matrix *321*
13.15 Polynomials of Matrices *322*
13.16 Functions of Matrices *322*
13.17 Representation of a Function of a Matrix by a Polynomial *322*
13.18 Sylvester's Theorem *323*

13.19 Solution of Algebraic Equations by the Use of Matrices *324*

Integration of Linear Differential Equations

13.20 The Matrizant *325*
13.21 Linear Equations with Constant Coefficients *326*
13.22 Linear Equations with Variable Coefficients; the Method of Mean Coefficients *327*

Applications of Matrices to Problems of Elastic Structures

13.23 The Principal Directions of Loading at a Point of an Elastic Body *328*
13.24 Applications to the Theory of Structures *330*

Applications of Matrices to Problems in Electricity

13.25 A-C Circuit Problems *332*
13.26 Three-phase Circuit Problems *333*
13.27 Four-terminal-network Theory *335*
13.28 Wave Propagation along a Cascade of Symmetrical Quadripoles *337*

Applications of Matrices to Vibration Problems

13.29 The Oscillations of Conservative Systems *340*
13.30 The Oscillations of Nonconservative Systems *343*
13.31 Conclusion *344*

14 Functional Transformations for Engineering Design **346**
BY JOHN L. BARNES

14.1 Introduction *346*

System-design Problems

14.2 Development of Automatic Bomber and Interceptor Systems *346*
14.3 Design of an Autopilot *347*
14.4 Typical Major Control Loop *348*

Functional Transformations

14.5 Functional Transformations for Phase 1 Design of Subsystems *349*
14.6 Functional Transformation for the System-analysis Problem *351*
14.7 Use of Integral Transformations *353*
14.8 System Synthesis *356*

14.9 Choice of Functional Transformation *357*
14.10 Synthesis of a Time-varying System *359*
14.11 Recent Advances *359*

15 Conformal Mapping Methods BY EDWIN F. BECKENBACH **361**

15.1 Introduction *361*
15.2 Cartography *362*
15.3 Conformality and Complex-variable Theory *368*
15.4 Laplace's Equation in Physics *372*
15.5 Existence Theorems *375*
15.6 Geometric Properties of Conformal Maps *379*
15.7 Special Functions *380*
15.8 Applications *384*
15.9 Construction of Conformal Maps *385*

16 Nonlinear Methods BY CHARLES B. MORREY, JR. **389**

16.1 Introduction *389*

Finite Systems of Equations

16.2 Newton's Method; Method of Perturbations; Modifications *390*
16.3 Remarks on Systems of Linear Equations *393*
16.4 Remarks on the Solution of Ordinary Differential Equations *395*
16.5 The Method of Continuity; Power-series Expansion *396*
16.6 Remarks concerning the Method of Steepest Descent *398*

Vector Notation

16.7 Vector Notation; Vector Operations; Length *399*
16.8 Differentiable Functions and Vector Functions *400*
16.9 A Sample Convergence Proof; an Inverse Mapping Theorem *402*

Normed Linear Spaces

16.10 Abstract Spaces *404*
16.11 Linear Spaces *404*
16.12 Normed Linear Spaces *406*
16.13 Completeness *407*

Functional Equations, Variational Problems, and Normed Linear Spaces

16.14 Scalar Functions; Functionals *408*
16.15 Simple Variational Problems *409*

16.16 Vector Equations in Normed Linear Spaces *413*
16.17 Remarks on Linear Equations *416*

Numerical Methods

16.18 Rayleigh-Ritz Method and Some Modifications for Variational Problems *416*
16.19 Remarks concerning the Rayleigh-Ritz Method and Finite-difference Methods for the Solution of Functional Equations *417*
16.20 Appendix *418*

17 What Are Relaxation Methods? BY GEORGE E. FORSYTHE **428**

17.1 Introduction *428*
17.2 Frameworks and Similar Problems *428*
17.3 Other Sources of Relaxable Systems of Equations *433*
17.4 The Variety of Methods of Solving Linear Systems *435*
17.5 Mathematical Analysis of Componentwise Relaxation *437*
17.6 Computing Modes and Frequencies *443*
17.7 Summary *444*

18 Methods of Steep Descent BY CHARLES B. TOMPKINS **448**

18.1 Introduction *448*
18.2 Multidimensional Coordinate Geometry and the Kaczmarz Method for Linear Equations *450*
18.3 Steep Descent and the Differential Equations of Steepest Descent for Functions Defined in Multidimensional Euclidean Space *455*
18.4 Remarks concerning Numerical Solution of Differential Equations of Steepest Descent; Desirability of Long Steps; the Hestenes-Stiefel Method of Conjugate Gradients *460*
18.5 Descent Subject to Constraints *463*
18.6 Generalization of Metric and Extension to Spaces of Infinitely Many Dimensions; Problems from the Calculus of Variations *466*
18.7 A Narrative Account of Mathematical Studies of the Points of Space for Which $J \leq c$ *473*
18.8 Conclusion *477*

19 High-speed Computing Devices and Their Applications BY DERRICK H. LEHMER **480**

19.1 Introduction *480*
19.2 Analogue Devices *481*

19.3 Digital Devices *485*
19.4 Matrix Problems *490*
19.5 Eigenvalue Problems *493*
19.6 Discrete-variable Problems *494*
19.7 Remarks on Numerical Analysis *495*

Name Index **499**

Subject Index **503**