
Contents

List of symbols	17
General symbols	17
Symbols defined in the text	17
Introduction	19
Part I. Selfadjoint operators in Hilbert spaces	25
1. Preliminaries	26
1.1. Fundamental concepts	26
1.1.1. Banach spaces and Hilbert spaces	26
1.1.2. Borel measures and integrals	27
1.1.3. Lebesgue decomposition of Borel measures	29
1.2. Vector-valued functions and vector measures	29
1.2.1. Borel functions, measurable functions and other types of functions.	29
1.2.2. Bochner integral	30
1.2.3. Banach and Hilbert spaces of vector-valued functions	30
1.2.4. Vector measures	32
1.2.5. Vector-valued Cauchy-Stieltjes integrals	33
1.3. Linear operators.	35
1.3.1. Elementary properties	35
1.3.2. Operator-valued functions	35
1.3.3. Selfadjoint, unitary and other special operators	35
1.3.4. Compact operators.	36
1.3.5. Spectra and resolvents	38
1.3.6. DUNFORD's functional calculus	38
1.4. C^* - and W^* -algebras	39
1.4.1. $*$ -algebras	39
1.4.2. Commutant and equivalence of projections	40
2. Multiplicity theory	41
2.1. Simple algebras and algebras with multiplicity	41
2.1.1. Simple algebras, simple and disjoint projections	41
2.1.2. Central covers.	42
2.1.3. Algebras and projections with multiplicity	42

2.2.	Algebras of type I, decomposition theorem	43
2.2.1.	Algebras of type I	43
2.2.2.	Abelian v. Neumann algebras	43
3.	Spectral theory	45
3.1.	Spectral measures, scalar spectral integrals and the spectral theorem . . .	45
3.1.1.	Spectral measures on Borel spaces and the Banach space $L^\infty(\Gamma, E)$. . .	45
3.1.2.	The spectral integral and the v. Neumann algebra $T(L^\infty)$	45
3.1.3.	The spectral theorem for bounded normal operators	46
3.1.4.	The spectral theorem and the functional calculus for unbounded selfadjoint operators	47
3.1.5.	Spectral integrals and Bochner integrals	48
3.1.6.	Spectral measures and resolvents	49
3.2.	Unitary evolution groups	50
3.2.1.	Stone's theorem	50
3.2.2.	Operator Fourier transformations	50
3.2.3.	The v. Neumann algebra generated by e^{-itH}	51
3.3.	Classification of selfadjoint operators and their spectra by properties of their spectral measures	52
3.3.1.	The spectrally discrete subspace	52
3.3.2.	The spectrally continuous subspace	52
3.3.3.	The singularly continuous subspace	53
3.3.4.	The absolutely continuous subspace	53
3.3.5.	The essential spectrum	54
3.3.6.	The spectral core	54
3.4.	Spectral properties of evolution groups and resolvents, and "sandwiched" resolvents	55
3.4.1.	Characterization of the spectrally discrete and the spectrally continuous subspaces.	55
3.4.2.	Convergence properties of the resolvent	56
3.4.3.	The "sandwiched" spectral measure	56
3.4.4.	The "sandwiched" resolvent	57
3.5.	Properties of the absolutely continuous subspace	58
3.5.1.	L^1 -properties and implications	58
3.5.2.	Some L^∞ - and L^p -properties ($p > 1$)	59
3.5.3.	L^2 -properties	61
3.5.4.	The absolutely continuous subspace for functions $\alpha(H)$	64
3.5.5.	Spectral manifolds	65
4.	Direct integrals and spectral representations	67
4.1.	Direct and maximal integrals	67
4.1.1.	Preliminaries	67
4.1.2.	Direct integrals	67
4.1.3.	Maximal integrals	68

4.2.	Direct sums and tensor products	69
4.3.	Direct integrals for separable admissible systems	70
4.3.1.	Generating sequences of functions	70
4.3.2.	Some properties of direct integrals with separable admissible systems	70
4.4.	The algebra of scalar operators and its commutant.	72
4.4.1.	Scalar operators	72
4.4.2.	The commutant of the scalar operators	73
4.4.3.	A special case	74
4.5.	Unitary invariants and spectral representations	75
4.5.1.	Spectral forms, the corresponding direct integrals, and spectral representations	75
4.5.2.	Simple operators	76
4.5.3.	Unitary invariants of an arbitrary bounded selfadjoint operator and the corresponding spectral representations	77
5.	Operator spectral integrals.	80
5.1.	Operator spectral integrals defined by Riemann-Stieltjes sums.	80
5.1.1.	Definitions and connections to scalar spectral integrals	80
5.1.2.	Existence of operator spectral integrals.	81
5.1.3.	Integration by parts	82
5.1.4.	An intertwining formula	83
5.2.	Vector spectral integrals for absolutely continuous spectral measures	85
5.2.1.	Step functions	85
5.2.2.	A general class \mathcal{F} of functions	86
5.2.3.	Vector spectral integral for functions from \mathcal{F}	87
5.2.4.	A spectral representation by vector spectral integrals	90
5.2.5.	Vector spectral integrals and Hilbert-Schmidt operators	92
5.3.	Approximate operator spectral integrals	92
	Notes and remarks to part I	93
	Part II. Algebras of asymptotic constants	95
6.	General theory of asymptotic constants	97
6.1.	Special limiting processes for vector- and operator-valued functions	97
6.1.1.	A class of equivalent limiting processes	97
6.1.2.	Strong, Abelian and absolute Abelian limits	101
6.2.	Definitions and general properties	104
6.2.1.	The definition of asymptotic constants	104
6.2.2.	Simple properties of the wave morphism	105
6.2.3.	Some properties of wave ideals	108

6.3.	Relations between the algebras $\mathcal{L}_\infty(\mathcal{H})$, $\mathcal{L}(\mathcal{H})$, $\text{dom } \mu_s^H$ and $\text{dom } \mu_a^H$	111
6.3.1.	Some technical criteria	111
6.3.2.	Wave ideals and compact operators	113
6.3.3.	The difference between $\ker \mu_s^H$ and $\ker \mu_a^H$	114
6.3.4.	Some relations between $\mathcal{L}(\mathcal{H})$ and $\text{dom } \mu^H$	115
6.4.	Topological characterizations of wave algebras	116
6.4.1.	Definitions of special topologies	116
6.4.2.	A dense subset in $\text{dom } \mu^H$	118
6.4.3.	Compatibility of the algebraic properties with the μ^H -topology.	119
6.5.	Generalizations	122
6.5.1.	Universal spectral projections	122
6.5.2.	Wave algebras with universal spectral projections	124
7.	Special classes of asymptotic constants	126
7.1.	Projections	126
7.1.1.	Spectral projections of selfadjoint asymptotic constants	126
7.1.2.	Projections P with $\mu^H(P) \neq \mu^{-H}(P)$	126
7.1.3.	Equivalence relations between projections	127
7.2.	Adjoint and partial isometries	129
7.2.1.	Adjoint	129
7.2.2.	Properties of partial isometries from $\text{dom } \mu^H$	130
7.2.3.	Unitary operators	132
7.3.	C^* - and W^* -subalgebras of the wave algebra	133
7.3.1.	μ^H as $*$ -homomorphism on $*$ -subalgebras	133
7.3.2.	μ^H as spatial isomorphism on W^* -subalgebras	134
7.3.3.	Functions of asymptotic constants	137
7.4.	Classes of asymptotic constants for special generators	139
7.4.1.	The map Γ_H and the wave ideal $\ker \mu_s^H$	139
7.4.2.	Multiplication operators	142
7.4.3.	Integral operators	143
7.4.4.	Integral operators with singularities of Cauchy type	145
8.	The invariance of wave morphisms and wave algebras	149
8.1.	The class of generators of a wave algebra or wave ideal.	149
8.1.1.	Some classes of generators	149
8.1.2.	On the class of generators of the wave algebra $\text{dom } \mu^H$	152
8.1.3.	The class of generators of the wave ideal $\mathcal{J} = \ker \mu_a^H$	153
8.2.	The invariance of the wave morphism μ_a^H	158
8.2.1.	Admissible functions	158
8.2.2.	The invariance of the wave morphism μ_a^H	161
8.3.	The invariance of the wave morphism μ_s^H	163
8.3.1.	A no-go theorem	163
8.3.2.	Some special classes of operators	164

Notes and remarks to part II	167
Part III. Two-space wave operators and scattering operators	169
9. Elementary Theory of wave and scattering operators	170
9.1. Pre-wave operators and their limits	170
9.1.1. Time-dependent and stationary pre-wave operators	170
9.1.2. Limits of pre-wave operators	171
9.1.3. Simple properties of the limit J_∞ of the pre-wave operator	173
9.2. Two-space wave operators	175
9.2.1. Definition of wave operators with respect to universal spectral projections	175
9.2.2. Completeness and semicompleteness	177
9.2.3. One-dimensional perturbations	178
9.3. Asymptotically equivalent identification operators	181
9.3.1. Simple properties of asymptotically equivalent identification operators	181
9.3.2. Criteria for completeness and semicompleteness	182
9.4. Wave operators and limits of pre-wave operators	183
9.4.1. Connection between W_+ and Ω_+^w	184
9.4.2. Asymptotic equivalence of Ω_+^w and J	184
9.5. Partially isometric wave operators	185
9.5.1. Asymptotically partially isometric identification operators	185
9.5.2. Completeness criteria	186
9.6. Scattering operator and scattering matrix	187
9.6.1. Scattering operator	187
9.6.2. Scattering matrix	188
9.7. Examples	190
9.7.1. Finite-dimensional perturbations	190
9.7.2. Special symmetric hyperbolic systems (uniformly propagative systems)	192
10. Identification operators	195
10.1. Abstract multichannel scattering theory	195
10.1.1. Channel projections and channel Hamiltonians	195
10.1.2. Reaction channels and asymptotic completeness	197
10.1.3. Example: Multiparticle scattering	199
10.1.4. Identification operator	200
10.1.5. General channel systems	202
10.2. Algebraic scattering theory	204
10.2.1. Basic ideas and notions	204
10.2.2. Algebraic scattering systems	205
10.2.3. General properties of algebraic scattering systems	207
10.2.4. Algebraic scattering systems and wave operators	208
10.2.5. Examples	209
10.3. Equations of second order in d/dt	213
10.3.1. Reduction to the Schrödinger equation	213
10.3.2. Connection between $U(t)$ and e^{-itB}	214

10.3.3.	The first identification operator	215
10.3.4.	The second identification operator	217
10.4.	Quantum field theory	220
10.4.1.	Abstract quantum fields	220
10.4.2.	Tensor algebra of test functions, Poincare group, and regular representation	223
10.4.3.	Quantum fields	225
10.4.4.	Example: The free scalar field of mass $m_0 > 0$	228
10.4.5.	Identification operators between free and interacting fields	230
10.4.6.	Wave operators for quantum fields	232
11.	Structural properties of wave and scattering operators.	233
11.1.	Structure of wave operators.	233
11.1.1.	Structure of wave operators for arbitrary universal projections	233
11.1.2.	The special cases $P_H = P_H^c$ and $P_H = P_H^{ac}$	234
11.1.3.	Structure of H_0 -semicomplete partially isometric wave operators.	235
11.1.4.	Structure of complete partially isometric wave operators	236
11.2.	Structure of scattering operators (the inverse problem of scattering theory)	238
11.2.1.	Asymptotic partial isometry of identification operators	238
11.2.2.	Formulation of the inverse problem	240
11.2.3.	The key problem	240
11.2.4.	Solution of the inverse problem	242
11.2.5.	Description of all solutions	243
11.2.6.	The inverse problem for multichannel identification operators	245
11.3.	The invariance principle for wave operators	245
11.3.1.	Formulation of the invariance principle	246
11.3.2.	Invariance principle, strong form	246
11.3.3.	Invariance principle, weak form	247
11.3.4.	A counterexample and piecewise linear functions	248
12.	Lax-Phillips evolutions and two-spaces wave operators	250
12.1.	Lax-Phillips evolutions.	250
12.1.1.	Incoming and outgoing subspaces	250
12.1.2.	Examples	250
12.1.3.	Structure theorem for Lax-Phillips evolutions	251
12.1.4.	Spectral representation of Lax-Phillips evolutions	254
12.1.5.	The Lax-Phillips scattering operator	254
12.2.	Two-space formulation of the Lax-Phillips scattering theory	254
12.2.1.	Construction of special incoming and outgoing translation representations	255
12.2.2.	Wave operators and scattering operator	256
12.3.	Analytic properties of the Lax-Phillips scattering matrix	258
12.3.1.	Analyticity in the upper half plane.	258
12.3.2.	Analytic continuation of $\hat{S}(z)$ into the lower half plane and the Lax- Phillips semigroup.	260

13.	Stationary theory	264
13.1.	The stationary pre-wave operator	264
13.1.1.	Integral representations	264
13.1.2.	Some properties of the stationary pre-wave operators	267
13.2.	Limits of the stationary pre-wave operator	271
13.2.1.	Weak limits	271
13.2.2.	Strong limits	274
13.3.	Stationary theory of the Abel wave operator in terms of resolvents only	276
13.3.1.	Stationary characterization of the Abel wave operator	276
13.3.2.	The wave matrix	278
13.4.	Stationary theory of the Abel wave operator using operator spectral integrals	280
13.4.1.	The general case.	281
13.4.2.	The case of partially isometric Abel wave operators	281
13.4.3.	Convergence to the wave matrix.	283
13.5.	A completeness criterion	284
13.5.1.	A property in connection with the adjoint of the wave operator	285
13.5.2.	Extension of the operator Z_+ and formulation of the criterion	289
	Notes and remarks to part III	291
	Part IV. Existence and completeness of wave operators	299
14.	Stationary methods	301
14.1.	Stationary methods with auxiliary manifolds	301
14.1.1.	Spectral forms and spectral manifolds	301
14.1.2.	Approximate spectral forms and auxiliary manifolds	302
14.1.3.	Existence and completeness of wave operators	304
14.1.4.	Existence of strong wave operators	310
14.2.	Factorization method	314
14.2.1.	The perturbation V and auxiliary manifolds	314
14.2.2.	Existence of wave operators under factorization assumptions	316
14.2.3.	Completeness of wave operators under smallness and compactness assumptions	317
14.3.	Applications	319
14.3.1.	Multiplication operator perturbed by integral operators (small gentle perturbations).	319
14.3.2.	One-dimensional perturbation	321
15.	Time-falloff methods	322
15.1.	Existence of wave operators	322
15.1.1.	Some abstract criteria	322
15.1.2.	Application: Short range perturbations of the Laplacian	325

15.1.3.	Application: Schrödinger operators with hard core potential	326
15.1.4.	Application: Schrödinger operators with rapidly oscillating potentials	327
15.2.	Completeness of wave operators	330
15.2.1.	Completeness from spectral assumptions on H	330
15.2.2.	Completeness by time-falloff conditions.	331
15.2.3.	Application: Differential operators of first order	337
15.2.4.	Application: Potential scattering	338
15.3.	An invariance principle.	340
16.	Trace class methods	343
16.1.	Wave operators for trace class perturbations	343
16.1.1.	A general theorem about trace class perturbations	343
16.1.2.	Other trace class results	346
16.2.	A stationary proof of a trace class theorem	347
16.3.	Applications	351
16.3.1.	Wave operators for $H = H_1 + H_2$, $H_0 = H_1 \oplus H_2$	351
16.3.2.	Wave operators for Dirac operators	353
16.3.3.	Wave operators for a weakly uniformly propagative system.	354
17.	Smooth perturbations	356
17.1.	Smooth operators	356
17.1.1.	Basic notions	356
17.1.2.	Representation as integral operators	361
17.1.3.	Local smoothness	364
17.2.	Existence of wave operators	367
17.2.1.	Symmetrical case	367
17.2.2.	Smallness condition	371
17.3.	Applications	373
17.3.1.	Smooth perturbations of $-i(d/dx)$	373
17.3.2.	Potential scattering	374
	Notes and remarks to part IV	374
	Part V. Some properties of the scattering operator, the scattering matrix, and the scattering amplitude	381
18.	Representations of the scattering operator and the scattering amplitude, and analyticity properties of the scattering amplitude	382
18.1.	General formulas and properties for S and $\hat{T}(\lambda)$ within the framework of the stationary theory	382
18.1.1.	Scattering amplitude and total scattering cross section	382
18.1.2.	A representation of the scattering operator by a spectral integral	382
18.1.3.	A general formula for $\hat{T}(\lambda)$	386

18.1.4.	An explicit formula for $\hat{T}(\lambda)$	387
18.1.5.	An application to potential scattering	391
18.2.	Smoothness conditions and the scattering amplitude	391
18.2.1.	Some formulas for the scattering amplitude	391
18.2.2.	A special case in the one-space theory	393
18.3.	Analyticity properties of the scattering amplitude and resonances	394
18.3.1.	General remarks.	394
18.3.2.	Analytic continuations of $\hat{T}(\lambda)$.	395
18.3.3.	An application to potential scattering	397
18.3.4.	The finite-dimensional Friedrichs model	398
19.	Spectral properties of the scattering amplitude	400
19.1.	Trace class conditions and scattering amplitude	400
19.1.1.	Calculation of the scattering amplitude for trace class perturbations	400
19.1.2.	A trace norm estimate of the scattering amplitude	402
19.1.3.	Example (potential scattering)	404
19.1.4.	The spectral shift and the trace formula	405
19.1.5.	Phase shift and spectral shift	407
19.2.	Time-falloff conditions and scattering amplitude.	410
19.2.1.	Spectral properties of parts of $S - P_0^{\text{ac}}$	410
19.2.2.	The scattering amplitude for "locally" compact perturbations	413
19.2.3.	The scattering amplitude for "locally" Hilbert-Schmidt perturbations	415
19.2.4.	The scattering amplitude for a special scattering system	415
Notes and remarks to part V		417
Bibliography		420
Books and monographs		420
Articles		422
Subject index		447