

Contents

<i>Introduction</i>	1
PART ONE THE BOUNDARY-VALUE PROBLEM WITHOUT SINGULARITIES	
<i>I Particular solutions of the system without singularities</i>	13
§ 1. Preliminary Remarks and Notation	13
§ 2. A Fundamental System of Solutions With a Prescribed Behavior in the Vicinity of Zero	14
§ 3. The Special Solution and Transformation Operator	20
§ 4. A Fundamental System of Solutions With a Prescribed Behavior at Infinity for the Case $\lambda \neq 0$	27
§ 5. A Fundamental System of Solutions With a Prescribed Behavior at Infinity for $\lambda=0$	34
<i>II The spectrum and scattering matrix for the boundary-value problem without singularities</i>	37
§ 1. The Point Spectrum	37
§ 2. Properties of the Matrix $E^{-1}(\lambda)$	42
§ 3. The Scattering Matrix	46
§ 4. Behavior of the Matrix $E^{-1}(\lambda)$ in the Neighborhood of $\lambda=0$	51
<i>III The fundamental equation</i>	57
§ 1. Derivation of the Fundamental Equation	57
§ 2. Properties of the Kernel	62
§ 3. Lemmas on Integral Equations With Kernels Dependent on a Sum	70
§ 4. Existence of Solutions	76
§ 5. Investigation of Homogeneous Equations Constructed from the Scattering Data	84

<i>IV Parseval's equality</i>	89
§ 1. Preliminaries	89
§ 2. Derivation of Parseval's Equality from the Fundamental Equation	94
§ 3. Derivation of the Fundamental Equation from Parseval's Equality	99
 <i>V The inverse problem</i>	 105
§ 1. Statement of the Problem	105
§ 2. Estimates for the Matrix $K(x,y)$	107
§ 3. Existence of the Derivatives of $K(x,y)$	110
§ 4. Derivation of the Differential Equation	117
§ 5. Fulfillment of the Boundary Condition	123
§ 6. Characteristic Properties of the Scattering Data and Scattering Matrix	129
§ 7. Examples	138
 PART TWO THE BOUNDARY-VALUE PROBLEM WITH SINGULARITIES	
 <i>VI Special transformation operators</i>	 147
§ 1. Method of Investigation	147
§ 2. Transformation Operators for Matrix Equations	153
§ 3. Transformation of Parseval's Equality	157
 <i>VII Spectral analysis of the boundary-value problem with singularities</i>	 163
§ 1. Statement of the Problem. Notation	163
§ 2. Particular Solutions	164
§ 3. The First Transformation	169
§ 4. The Second Transformation	177
§ 5. The Third Transformation	188
§ 6. Properties of the Scattering Data (the Case $a_{22}=0, a_{12} \neq 0$)	208
§ 7. Behavior of the Scattering Matrix When $\lambda \rightarrow 0$. Summary of Results	217

<i>VIII Reconstruction of the singular boundary-value problem from its scattering data</i>	223
§ 1. Case (a)	223
§ 2. Case (b)	226
§ 3. Case (c)	234
§ 4. Algorithm for Determining the Potential Matrix. Examples	254
 <i>Appendix I On the characteristic properties of the scattering data of the boundary-value problem without singularities</i>	265
§ 1. Factorization of a Unitary Matrix	265
§ 2. Indices of $S(\lambda)$	270
§ 3. A New Characterization of the Scattering Data	280
 <i>Appendix II Refinement of certain inequalities</i>	283
 <i>Bibliography</i>	287
 <i>Index</i>	289
 <i>Index of notations</i>	291