

Contents of Volume 1

	Pre	face		xvii
1	Intr	oductio	on	1
	1.1	The p	place of symmetry in physics	1
	1.2	-	ples of the consequences of symmetry	3
		1.2.1	One particle in one dimension (classical)	3
		1.2.2	One particle in two dimensions (classical)	3
			Two particles connected by springs (classical)	4
		1.2.4		
			mechanics—spherical symmetry and degeneracies	5
		1.2.5		
			mechanics—parity and selection rules	6
		1.2.6		
			physics	7
	1.3	Sumn	nary	8
2	Gro	ups an	d Group Properties	9
	2.1		ition of a group	9
	2.2		ples of groups	11
	2.3		orphism	16
		Subgr		17
	2.5	_	lirect product group	17

vi Contents

	2.6	Conju	igate elements and classes	18	
	2.7	Exam	ples of classes	19	
			The rotation group \mathcal{R}_3	19	
			The finite group of rotations D_3	20	
			The symmetric group \mathscr{S}_3	21	
	2.8	The c	lass structure of product groups	21	
	2.9		roup rearrangement theorem	22	
	Bibl	iograph		22	
	Pro	blems		22	
3	Line	ar Alg	ebra and Vector Spaces	24	
	3.1	Linea	r vector space	25	
	3.2 Examples of linear vector spaces		ples of linear vector spaces	27	
		3.2.1	Displacements in three dimensions	27	
		3.2.2	Displacement of a set of N particles in three		
			dimensions	28	
		3.2.3	Function spaces	28	
		3.2.4	Function space with finite dimension	29	
		3.2.5	Wave functions	29	
	3.3	Linea	r operators	30	
	3.4		nultiplication, inverse and transformation of operators	32	
	3.5				
		operators			
	3.6				
	3.7				
	3.8	Exam	ples of linear operators	38	
			Rotation of vectors in the xy-plane	38	
		3.8.2	Permutations	39	
			Multiplication by a function in function space	39	
			Differentiation in function space	40	
		3.8.5	Induced transformation of functions	40	
		3.8.6	Further example of induced transformation of		
			functions	41	
		3.8.7	Transformed operator	41	
	Bibliography			42	
		blems	•	42	
4	Gro	up Rep	presentations	43	
	4.1	Defin	ition of a group representation	43	
	4.2	Matrix representations		44	
	4.3	8 Examples of representations		45	
		4.3.1	The group D_3	45	
		4.3.2	The group \mathcal{R}_2	46	
		4.3.3	Function spaces	47	
	4.4			48	
	4.5		ucibility	50	
	16	Eavi	volent representations	52	

	Contents	vii
	4.6.1 Proof of Maschke's theorem	53
4.7	Inequivalent irreducible representations	54
4.8	Orthogonality properties of irreducible representations	54
	4.8.1 Proof of Schur's first lemma	58
	4.8.2 Proof of Schur's second lemma	60
4.9	Characters of representations	60
	representations	61
4.11	Use of group characters in reducing a representation	62
4.12	A criterion for irreducibility	63
4.13	How many inequivalent irreducible representations?—The	
	regular representation	64
		66
4.15	Construction of the character table	67
4.16	Orthogonality of basis functions for irreducible	
	representations	68
		70
4.18		
	* ·	73
		74
4.20		
		78
		81
		83
		83
		85
		85
		89
		90
		91 92
		93
3.0		93
		95 95
		96
	, , , , , , , , , , , , , , , , , , , ,	96
57		97
		99
5.0		100
		101
5.9		102
		103
		104
	· · ·	104
		106
	4.8 4.9 4.10 4.11 4.12 4.13 4.14 4.15 4.16 4.17 4.18 4.19 4.20 4.21 Bibli Prob Sym 5.1 5.2 5.3 5.4 5.5 5.6 5.7 5.8	 4.6.1 Proof of Maschke's theorem 4.7 Inequivalent irreducible representations 4.8 Orthogonality properties of irreducible representations 4.8.1 Proof of Schur's first lemma 4.8.2 Proof of Schur's second lemma 4.9 Characters of representations 4.10 Orthogonality relation for characters of irreducible representations 4.11 Use of group characters in reducing a representation 4.12 A criterion for irreducibility 4.13 How many inequivalent irreducible representations?—The regular representation 4.14 The second orthogonality relation for group characters 4.15 Construction of the character table 4.16 Orthogonality of basis functions for irreducible representations 4.17 The direct product of two representations 4.18 Reduction of an irreducible representation on restriction to a subgroup 4.19 Projection operators 4.20 Irreducible sets of operators and the Wigner – Eckart theorem 4.21 Representations of direct product groups Bibliography Problems Symmetry in Quantum Mechanics 5.1 Brief review of the framework of quantum mechanics 5.2 Definition of symmetry in a quantum system 5.3 Degeneracy and the labelling of energies and eigenfunctions 5.4 Selection rules and matrix elements of operators 5.5 Conservation laws 5.6 Examples 5.6.1 Symmetry group C₃ 5.6.2 Symmetry group D₃ 5.6.3 Symmetry group S₂ 5.6.4 Symmetry group P₂ 5.6.4 Symmetry group P₃ 5.6.5 Symmetry group P₂ 5.6.4 Symmetry group P₃ 5.6. Symmetry group of group theory in a variational approximation

viii Contents

	6.1	The harmonic approximation	107		
	6.2	Classical solution	108		
	6.3	Quantum mechanical solution			
	6.4	Effects of symmetry in molecular vibrations 11			
	6.5	Classification of the normal modes	113		
		6.5.1 The water molecule	115		
		6.5.2 The ammonia molecule	116		
	6.6	Vibrational energy levels and wave functions	117		
	6.7	Infrared and Raman absorption spectra of molecules	120		
		6.7.1 Infrared spectra	120		
		6.7.2 Raman spectra	121		
	6.8	Displacement patterns and frequencies of the normal modes	122		
	Bibl	iography	124		
		blems	124		
7	Con	tinuous Groups and their Representations, Including Details			
		of the Rotation Groups \mathcal{R}_2 and \mathcal{R}_3	125		
	7.1	General remarks	126		
	7.2	Infinitesimal operators	127		
	7.3		130		
		7.3.1 Irreducible representations	131		
		7.3.2 Character	131		
		7.3.3 Multiplication of representations	132		
		7.3.4 Examples of basis vectors	132		
		7.3.5 Infinitesimal operators	133		
	7.4	The group \mathcal{R}_3	134		
		7.4.1 Infinitesimal operators	135		
		7.4.2 Irreducible representations	137		
		7.4.3 Characters	140		
		7.4.4 Multiplication of representations	141		
		7.4.5 Examples of basis vectors	143		
		7.4.6 Irreducible sets of operators and the Wigner-Eckart			
		theorem	146		
		7.4.7 Equivalent operators	147		
	7.5	The Casimir operator	148		
	7.6	Double-valued representations	150		
	7.7	The complex conjugate representation	153		
	Bib	liography	153		
	Pro	blems	154		
8	Ang	Angular Momentum and the Group \mathcal{R}_3 with Illustrations from			
	Ato	mic Structure	156		
	8.1	Rotational invariance and its consequences	156		
	8.2	2 Orbital angular momentum of a system of particles 13			
	8.3				
	8.4	4 Intrinsic spin			
	0.5	The hydrogen atom	166		

Contents	İ¥
Contents	i A

	8.6	The structure of many-electron atoms	170
		8.6.1 The Hamiltonian	170
		8.6.2 The Pauli principle and shell filling	171
		8.6.3 Atoms with more than one valence electron— LS	
		coupling	173
		8.6.4 Classification of terms	176
		8.6.5 Ordering of terms	179
	Bibl	iography	181
		blems	181
9	Poir	nt Groups with an Application to Crystal Fields	183
	9.1	Point-group operations and notation	184
	9.2	The stereogram	184
	9.3	Enumeration of the point groups	186
		9.3.1 Proper groups	186
		9.3.2 Improper groups	191
	9.4	The class structure of the point groups	192
		9.4.1 Proper point groups	193
		9.4.2 Improper point groups	193
	9.5	The crystallographic point groups	196
	9.6	Irreducible representations for the point groups	197
	9.7	Double-valued representations of the point groups	199
	9.8	Time-reversal and magnetic point groups	201
	9.9	Crystal field splitting of atomic energy levels	202
		9.9.1 Definition of the physical problem	202
		9.9.2 Deduction of the manner of splitting from symmetry	
		considerations	204
		9.9.3 Effect of a magnetic field	209
	Bibl	iography	210
	Pro	blems	211
10	Isos	pin and the Group SU_2	213
	10.1		214
		10.1.1 Isospin labelling and degeneracies	215
		10.1.2 Splitting of an isospin multiplet	218
		10.1.3 Selection rules	221
	10.2	Isospin in elementary particles	222
		10.2.1 Collisions of π -mesons with nucleons	223
	10.3	Isospin symmetry and charge-independence	223
	Bibl	liography	224
	Pro	blems	224
11	The	Group SU ₃ with Applications to Elementary Particles	226
	11.1		227
	11.2		230
	11.3	-	231
	11.4		232
	11.5		233

x Contents

	11.6	Irreducible representations of SU_3	233			
		11.6.1 Complex conjugate representations	241			
		11.6.2 Multiplication of representations	242			
	11.7	Classification of the hadrons into SU_3 multiplets	243			
	11.8	The mass-splitting formula	244			
	11.9	Electromagnetic effects	247			
	11.10	Casimir operators	248			
	Biblio	Bibliography				
	Probl	lems	249			
12	Supermultiplets in Nuclei and Elementary Particles—the Groups					
	SU_4 and SU_6 and Quark Models					
	12.1	Supermultiplets in nuclei	252			
	12.2	Supermultiplets of elementary particles	255			
	12.3	The three-quark model	257			
	12.4 The nine-quark model					
	12.5	Charm	262			
	Biblio	Bibliography				
	Prob		263			
Apı	pendix	1 Character Tables for the Irreducible Representations	of			
E1		the Point Groups	265			
Ap	pendix		275			
Ind	ex to 1	Volumes 1 and 2 (adjacent to p. 280)	I			